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REPORT OF THE AMERICAN COMMISSIONERS OF THE INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS

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REPORT OF THE AMERICAN COMMISSIONERS OF THE INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS.

INTRODUCTION.

At the Fourth International Congress of Mathematicians, held at Rome in April, 1908, an International Commission on the Teaching of Mathematics was created. Representatives of the various leading countries were selected to carry on an investigation of the teaching of the subject throughout the world and to prepare reports before the meeting of the congress of 1912.

The American commissioners held their first meeting in New York City on March 26 and 27, 1909. It was resolved to form an advisory council to which might be referred important general questions and with the members of which the commissioners might consult collectively or individually as occasion might arise, and to invite the following gentlemen to membership: The United States Commissioner and ex-Commissioner of Education, the presidents of Harvard, Columbia, and Chicago Universities, and the presidents and ex-presidents of the American Mathematical Society and of the American Federation of Teachers of the Mathematical and the Natural Sciences. All of these gentlemen consented to serve, and to their earnest cooperation is due in large measure the success of the work. A list of the members of the advisory council is appended to this report.

Various committees and subcommittees were invited to consider and report upon special fields, in the spirit of the plan of the central committee. A list of the members of these committees, together with the plan of official publication of their reports, is given in the Appendix.

In a communication to the various committees at the time of the organization of the work the American commissioners said:

> The preparation of the reports calls for a comprehensive survey of our educational system in general and of the work in mathematics in detail; for a sketch of the unparalleled activities of recent decades in the development of existent institutions and in the genesis of new ones; for an account of the modifications in the work in mathematics that have already been made and of the reforms that are still under consideration. It affords us an opportunity to take an inventory of present conditions and to weigh the proposals that have been made for the future. The spirit of the work should therefore be judicial, not legislative.

Since the reports are destined for the educational public of many nations, it will be expedient to give a concise account of the salient features of our schools and the conditions under which we work, such as are usually tacitly assumed in our own discussions. Work in mathematics must be regarded and interpreted in the light of its environment, and our reports should furnish the readers of other nations with information respecting our educational system and conditions analogous to that which we shall expect from them.

America is unique in the liberty left to individual initiative in matters of education, and in the absence of authoritative central legislation and supervision. It is desirable, therefore, that the reports describe clearly the practical working of this freedom and its effect, good and bad, upon our progress in general and in mathematical education.

We likewise stand alone in the complete separation of church and state throughout our entire history, and an account of the effect of this on the work of education will be of interest to those nations that have faced or are facing serious problems in connection with religious instruction.

We are also unique in the brevity of our educational history, and in the consciousness of living in its formative epoch, a consciousness that permeates all our activities. While it is not the aim of the commission to study educational history or to tabulate statistics, it may be necessary, under American conditions, to do some work of this nature in order to understand the present and to forecast the future.

Although the great central problems are fully stated in the preliminary report, we have others that are peculiar to ourselves, such as the education of the negro and the training and Americanizing of the large number of uneducated immigrants constantly pouring in upon us.

In all of these respects the various reports should pay suitable attention to the international character of the work.

The topics considered by the various committees were as follows:

Committee I. General elementary schools.
Committee II. Special elementary schools.
Committee III. Public general secondary schools.
Committee IV. Private general secondary schools.
Committee V. The training of elementary and secondary teachers.
Committee VI. Technical secondary schools.
Committee VII. Examinations.
Committee VIII. Influences tending to improve the work of the teacher.
Committee IX. Technological schools of collegiate grade.
Committee X. Undergraduate work in colleges of liberal arts.
Committee XI. West Point and Annapolis.
Committee XII. Graduate work in universities and institutions of like grade.

The names of the members of these committees are appended to this report. Their several reports have been published by the Bureau of Education, have been transmitted to all foreign commissioners and to all bureaus of education in the various States of our country and the various countries abroad, and have been distributed freely to those who have applied for them.

THE REPORT OF THE AMERICAN COMMISSIONERS.

NATURE OF THE REPORT.

In the following general report the aim is to present a concise view of the organization, methods, and problems of the teaching of mathematics in the United States, touching particularly upon the larger questions that seem to merit especial emphasis, and referring to the special reports for details with respect to the various types of schools. The report considers the following general topics: I. General American Conditions; II. Mathematics in the Elementary School; III. Mathematics in the Secondary School; IV. Mathematics in the Higher Institutions of Learning.

I. GENERAL AMERICAN CONDITIONS.

THE CENTERS OF EDUCATIONAL AUTHORITY IN THE UNITED STATES.

In the United States of North America there are 48 States, each of which is self-governing, except in those items specially entrusted to the central Government by the Constitution. Among these items the formation and administration of an educational system is not found. Consequently there exists no national authority in educational matters in the United States, the largest unit of authority being the State. That there are not 48 widely different educational systems in the country, that a large measure of uniformity does exist in the educational work of the whole country, that it is possible to speak of a single educational system found (with local variations) throughout the whole country, is due simply to the homogeneity of thought and life of the country and not to any constitutional requirement. The local variations are, of course, important, but this report can not concern itself with them; here only a composite picture is possible, and it must be understood that only the general situation is set forth.

FINANCIAL SUPPORT OF AMERICAN EDUCATIONAL INSTITUTIONS.

Throughout the whole country educational institutions from the lowest to the highest are supported either by public funds raised by direct taxation by a State or one of its subdivisions (say a county or city), or by private funds. The private institutions are sometimes supported by philanthropic endowments supplemented by

tuition fees, and sometimes are commercial ventures in which the tuition fees received are expected to pay the entire cost of maintenance of the institution and suitable returns on the capital invested. Grants from public funds to private institutions are rarely made, and gifts from philanthropic individuals are made less frequently to public than to private institutions.

In the public institutions tuition is usually free to all residents of suitable age of the community supporting the school, and textbooks are often furnished free to all pupils.

SUPERVISION BY THE STATE.

The State supervises, at most, the institutions supported by public funds.[1] As no legal rights or privileges are conferred by graduation from schools of any of the types to be considered in what follows,[2] State approval would give no additional value to the diploma of a private institution, and such institutions have no incentive voluntarily to submit themselves to State inspection or control. Those who seek graduation from an educational institution in the United States do so for one or more reasons of the following nature: On account of the inherent value of the education obtained; to conform to custom; on account of the social and moral consequence given by such graduation; because the diploma is required for admission to another educational institution; for appointment as teacher or the like.

TYPES OF SCHOOLS.

Normally the pupil passes in order through the following types of schools: Entrance age, 3, 4, 5—Kindergarten; 6, 7, 8, 9, 10, 11, 12, 13—Elementary school; 14, 15, 16, 17—Secondary school; 18, 19, 20, 21—College or institution of collegiate rank; 22, 23, 24—University or institution of university rank.

The elementary schools.—The kindergartens are often private institutions. Their work is not prerequisite to entrance into an elementary school, and the school career of almost all children begins

[1] A full account of the system of State inspection is given in the report of Committee VIII.

[2] In some of the States legal rights are conferred by graduation from certain types of professional schools, as those of law, medicine, dentistry, pharmacy, etc., but as no mathematics is taught in these institutions they fall without the scope of the present report. Of course these institutions require students entering them to have reached a certain standard of general education varying widely with different institutions and conveniently certified by a diploma, and either legal enactment or general custom often requires graduation from a school of certain type for appointment as teacher, but private schools have hitherto had no difficulty in securing acceptance of their diplomas in fulfillment of these requirements on terms of equality with those of the State-inspected schools; so that these requirements do not exert any pressure upon private schools to seek State inspection. (In case such inspection were requested, it might be refused on the ground of expense.)

with the elementary school. The elementary schools are almost entirely public, and here nearly all the children of any given vicinity, rich or poor, are taught side by side on terms of equality for a period of eight years. In the Northern States negro children constitute only a small percentage of the total and are usually taught with the white children; in the Southern States they constitute a large percentage of the total, sometimes even more than half, and are taught separately. The school career of the great majority of the children ends with the completion of the elementary school or sooner.[1]

Sequence of schools.—Generally speaking, the completion of the work of an elementary school is required for admission to a secondary school; similarly, completion of the work of a secondary school is prerequisite for admission to an institution of collegiate rank, and finally the diploma of a collegiate institution is the basis of formal admission to university work. But there is no required sequence of types of institutions or of curricula. There is practically but one type of the elementary school, which is the common basis for all subsequent work. In the other institutions there are various types and curricula, but generally speaking it is possible to pass (with more or less supplementary work) from any type of secondary school to any collegiate institution and thence to any university, or to change from one type of institution or curriculum to another while passing through it. It must be noted that different institutions of the various cultural types (classical, modern languages, scientific) practically never exist, but the uniformly prevailing practice is to carry through in the same institution curricula of such of these types for which there is a sufficient demand. So far as these curricula cover common ground, the instruction is common, and if a pupil finds, as his mind and personality develop or as his aims in life become more definite, that another curriculum than the one which he has begun would fit his needs better, he can change from one to the other by making up such work of the new curriculum as he has not had. He may do this either by attendance upon class instruction or by satisfactory private instruction with examination. When made in a serious spirit such transfers are encouraged and facilitated by the school authorities. Additional time is usually required to complete the course, but in favorable cases the privileges of election may permit the change to be made without loss of time.

Separate institutions of technological, commercial, manual-training, and agricultural types exist, but such work is also often carried on in general institutions. When this is the case students taking one type of curriculum are not sharply differentiated from those taking another, and students taking different curricula will have more

[1] For a detailed account of the organization of educational institutions in the United States, see the report of Committee I.

or less instruction in common. One of the most prominent common subjects is mathematics. It is a matter of constant occurrence that the same class in mathematics will contain classical, literary, and scientific students, to whom mathematics is primarily a culture subject, side by side with engineering and other students, to whom mathematics is primarily a vocational tool. All these different types of students interact on each other within the classroom, and in their association outside the classroom all meet on common ground. In after life the slight distinction of differing curricula is soon effaced, and only the bond of a common education in the same institution remains. The teachers of the common subjects (for example, mathematics) feel themselves the teachers equally of all types of students that come under their instruction. The isolation of teachers and pupils in separate institutions—classical, literary, or scientific, with their separate traditions, aims, and esprit de corps—does not exist at all in the United States, and the separation of the vocational institutions from the cultural exists only to a limited extent.

THE PREPARATION OF TEACHERS.

One of the gravest problems of American instruction in mathematics, from the lowest to the highest, is that of the adequate preparation of the teachers. This is, of course, more or less of a problem everywhere, but the peculiar difficulty of American conditions will appear from certain statistics.

During the past half century, say from 1860 to 1910, a period in which free public education has become general, together with the compulsory feature, our population has substantially trebled, increasing from 31 millions to 92 millions. At present nearly 18 million pupils (almost 20 per cent of our population) are enrolled in our elementary schools. In the same half century the annual number of immigrants has risen from 150,000 in 1860 to over a million in 1910, and new States have been admitted to the Union representing over a million square miles of territory, resulting in a vast amount of moving about the country. In the 20 years from 1890 to 1910 our exports rose from 858 millions of dollars a year to 1,745 millions, and our imports from 789 millions to 1,558 millions, and our manufactures of iron and steel, for example, rose from 122 millions to 179 millions.

The bearing of all this upon our school problem is very evident. Suddenly, within less than two generations, a nation has been confronted with the demand for universal education. This demand would be serious enough with a population that was static as to numbers or static as to residence; but when the population has been multiplied by three, when children have been continually changing

from place to place, when the school has had to teach not only mathematics but also conversational English to the children of a million immigrants a year, when the country has had not only to maintain its schoolhouses on the original territory but to provide for a million square miles besides, and when the increase in trade, in manufacture, and in wealth in general has been such as to draw its most active men into business, the solution of the educational problem has not been a simple one. Teachers have had to be supplied for these elementary schools. It has not been a time for standing on technicalities of training, nor even for demanding a high degree of culture; it has rather been a period for immediate action, for avoiding at all hazards the perils of illiteracy, and for maintaining the public school at whatever grade of efficiency was attainable, even though not in general satisfactory. Since the best type of men could not be secured in any considerable number, owing to the financial opportunities offered by a new country, since teaching was one of the few financial openings for women, and since in the earlier school years the work of the woman is more satisfactory than that of the man, there has come about a state of affairs not to be found in any of the older countries. To-day four-fifths of the teachers in our elementary schools are women,[1] and only a relatively small number remain in the profession more than a few years. The problem of training such an army of women teachers, most of whom remain in the schools but a relatively short time, has been and is one of great difficulty, and its influence upon American education in general and upon elementary mathematics in particular is serious.

THE TRADITIONS OF OUR SCHOOLS.

The elementary schools of the United States inherited the mathematics of the English schools of similar grade. In our colonial period this consisted solely of arithmetic, with such mensuration as naturally entered into this subject. The arithmetic was generally intended to be utilitarian, although it still retained a little of the ancient theory, such as cube root, progressions, and the Euclidean method of finding the greatest common divisor. It early became the custom to study arithmetic in the first four school years from a primary textbook, in the next three or four years from a higher one, and often during the next two or three years from one of a still more difficult grade. The idea that arithmetic has a high culture value was generally accepted; and even if the problems were not very practical, they were felt to be valuable.

Given the conditions under which our schools were expanding, it would have been difficult to break away from any established tradi-

[1] In 1907-8 there were 391,939 women out of a total of 496,612 teachers in the elementary schools.

tions, whether good or bad. The man battling with a flood must struggle for his life; he can not be carrying out reforms. Hence it came about that while courses of study were changing in the older countries and a high grade of scholarship was entering into the elementary schools and children were remaining in a single school long enough to allow a reform to be tried out, or were in a country where centralized authority could plan such reform for all schools of similar character, in the United States the great problem was to keep even what we had inherited. The difficulty of securing well-trained teachers early led to the development of an unusually good type of textbook—one that might compensate for the lack of skill on the part of the teacher, and this result, fortunate though it was, tended still more to keep the traditional course intact. As a result the general nature of the American curriculum in elementary and secondary mathematics has changed less in a century than that of several of the older countries.

SUGGESTIONS FOR CHANGE.

In a new country, with few overpowering traditions, the number of suggestions for change is bound to be very great, and usually to be correspondingly ill-considered. Such has been and is the case with us. It is probably well within the facts to assert that more experiments in the teaching of elementary mathematics have been given a limited trial in the United States, and fewer of these for sound reasons, or in an adequate manner, than in any other prominent country in the world. Conditions already mentioned have brought this about—the independence of States, of cities, and of villages, the lack of any large body of scientifically trained teachers, the lack of scholarly supervision, the fact that the sexes are so unevenly distributed in the teaching force—all these have encouraged superficial experiments but have not generally made for thorough scholarship and persistent investigation.

While the difficulties in the way of some haphazard trial of new ideas in single institutions or localities are at a minimum in the United States, this very freedom makes the general introduction of any desirable improvement in the United States exceptionally difficult. General change can be brought about only by the independent action of hundreds or thousands of governing bodies, and the failures of even a few ill-considered and local experiments, of which the most erratic often gain most notoriety, tend to make those governing bodies hesitate to authorize any experiments whatever, no matter how well-considered and promising.

Taken as a whole the country is very conservative in the matter of change. It may appear to be teeming with experiments, but one school striving to pioneer a new and supposedly better road attracts

more attention than a thousand quietly following the beaten path. To introduce a change generally throughout the country requires not only that the same question be argued out in a thousand independent forums, but also that in each the spirit of conservatism be overcome.

In the elementary schools experiments have been decidedly more numerous and more varied than in the secondary schools. There are obvious reasons for this. The elementary school takes its pupils without preparation; the overwhelming majority of them receive no education beyond the elementary school; and the next following type of school, the secondary school, exercises little influence on the work of the elementary school.[1]

The secondary school, on the other hand, must receive its pupils at the stage at which the elementary school turns them out, and the next following type of institution, the college, exerts a determining influence on the curriculum and work of the secondary school.[2]

Since the equipment with which pupils are received and that which they must have on leaving are fixed by conditions practically beyond the control of any single secondary school, very definite limits are placed upon the possible range of experiments and the character of changes.

Further, experiment or change means disturbance, temporarily increased difficulty for teachers in their work of instruction, perhaps more or less dissatisfaction with the innovation on the part of the pupils and their parents, with the attendant possibility of loss of pupils; and private schools at least, which technically are the most independent of all, would look askance at a plan involving such a possibility.

Still further, the great majority of secondary schools, acting singly, justly feel that their work is, primarily, teaching according to the existing standards, and not the initiation of educational reforms; that their governing bodies and their teaching staff lack the time and ability, the training and the breadth of view, necessary to deal adequately with the general questions involved.

Consequently, educational experiments in the secondary field are by no means as frequent or as effective as the complete freedom to undertake them might lead one to expect.

Even among the States that have a strong central administration none has, as a matter of fact, ventured to put into force an educational program varying materially from those current in its sister States.

So that a country which in commerce is so alert and progressive, whose men of business are constantly replacing the "good enough"

[1] See the report of Committees I, II, III, and IV. [2] See the report of Committee X.

by the "better," is in educational matters among the most conservative of the world's leading nations, generally moving along lines of least resistance, frequently clinging to the "good enough" because the way to the "better" is not authoritatively pointed out.

It is worth while to consider the question whether it would not be possible and desirable to make a more systematic study of the problems relative to the teaching of mathematics in this country than has hitherto been undertaken, in such wise that the results would command national confidence sufficiently to assure them a certain moral authority as standards.

II. MATHEMATICS IN THE ELEMENTARY SCHOOL.

General status.—As stated earlier in this report, the elementary school in the United States usually has a course of eight years beyond the kindergarten, the latter being an optional course found chiefly in cities. Each State has the power to lengthen or shorten this course, but in general eight years may be taken as the norm. These years are usually designated as *grades*, the first grade usually being entered at the age of 6 years, the second at the age of 7, and so on. Nearly all public elementary schools are coeducational. Provision is often made for the omitting of a grade by a pupil who is capable of proceeding more rapidly than the average. This is done in various ways, as by half-year promotions, and by having special classes for the purpose. Some schools promote their pupils by subjects, instead of by grades. None of these questions as to how the promotion is effected, however, has a direct bearing upon the work in mathematics.

In general the pupil knows something about the lower numbers when he enters school. The work in the first year varies greatly throughout the country. In some places no arithmetic is taught as such in the first grade, but an attempt is made to introduce it incidentally in connection with other work. In other places a course in formal arithmetic is seriously undertaken. In New York State, for example, the official curriculum requires the study of numbers to 100, the memorizing of the 45 combinations in addition, and the process of carrying in the adding of numbers.

In the same way the work in the other grades varies in different parts of the country, the details of which are set forth in the report of Committee I. For the sake of readers who wish a mere summary, however, the following outline of the New York State curriculum, adopted in 1910, will show the general nature of the work in the elementary schools of the United States:

Grade I. Age 6–7 years. Number, space to 100 and the addition table. The process of carrying in addition.

Grade II. Age 7–8 years. Subtraction. The multiplication table to 9 × 9. Process of carrying in multiplication by a one-figure multiplier.

Grade III. Age 8–9 years. Multiplication table to 12 × 12. Multipliers of more than one figure. Long division. Measures.

Grade IV. Age 9–10 years. Common fractions, including all operations. Measures, including all of the common tables.

Grade V. Age 10–11 years. Decimal fractions. Denominate numbers.

Grade VI. Age 11–12 years. Review of common and decimal fractions. Percentage begun, with applications to profit and loss, commission, and simple interest.

Grade VII. Age 12–13 years. Percentage continued, with applications to simple and compound interest. Simple banking business. Ratio and proportion. Square root.

Grade VIII. Age 13–14 years. Review of measures. Applications to measurement of common surfaces and solids. Rapid calculation. Stocks and bonds, insurance, taxation. Algebraic notation sufficient for the use of formulas. A special class in algebra is also permitted for Grade VIII.

It is possible that this course is too difficult in the early grades. In many American schools the multiplication table is learned only to 10 × 5 in Grade II, the rest of it being postponed to Grade III. There has been a tendency in some sections of the country to eliminate formal arithmetic in Grade I and even in Grade II, no number work being performed there except as it arises incidentally in connection with the other work of the children. In the above course, neither of these views of education is recognized. The course requires the multiplication table in Grade III to 12 × 12, which is more than is demanded generally in this country. The completion of long division in Grade III is also more than is commonly required in other parts of the country. Nevertheless the course is considered a strong one, and it represents a reaction against a recent tendency to relieve children from serious effort in obtaining an education.

Methods commonly employed.—In general, in the United States, a textbook is placed in the hands of the pupil in the third grade. Textbooks have also been prepared for the second grade. These textbooks usually give some explanation of the various processes, and supply a sufficient list of problems to give the child a mastery of each. Of late the tendency has been to give less in the way of explanation in the textbook, the increasing efficiency of the teaching force permitting this work to be left for the instructor. This has permitted of more problems in the textbook, thus bringing them to resemble the European collections of exercises to a greater extent than was formerly the case. In general, the nature of the problems has changed satisfactorily with the change in business customs and in the prices of commodities, and there is less cause for complaint on account of obsolete material than was the case a few years ago.

The teacher usually explains a new process by means of questions and answers, and when it seems understood a number of problems

are given to be worked out for the next recitation. In the lower grades little or no home work is assigned, the work being done during a study hour in school. In the upper grades home work is frequently assigned.

IMPROVEMENTS IN ARITHMETIC.

Agitation for improvement.—The question of improving the work in arithmetic has been much agitated during recent years, and this agitation has led to several good results. In the first place, the past quarter of a century has seen a weeding out of most of the obsolete applications of arithmetic. To-day it must be said for the subject that a large per cent of the problem material represents modern conditions of life, and is of a sufficiently varied character to meet the reasonable needs of all classes. There is a general agreement that the school should not attempt to instruct in the technicalities of particular trades or vocations, but should give a sufficiently wide range of the applications of arithmetic to prepare the pupil to master these technicalities for himself when he enters upon his life work. This the school is doing as well as the crowded curriculum and other conditions will permit.

A second improvement of great importance has resulted from the consideration of child psychology. Apart from details of no particular significance, one feature stands out prominently—that the subject matter of arithmetic is better arranged than formerly to arouse the interest and to meet the immediate needs of a child. A number of experiments in this line have been tried, and as a result we seem to be tending toward a still more satisfactory solution of the problem of arrangement of material.

A third point worthy of mention is the growing recognition of the fact that no textbook can meet all local conditions with respect to appropriate material for problems. Teachers are recognizing the value of themselves securing practical problems that represent the industries of their respective localities, both for the interest that they have for the pupils and for their value in the life of the community.

Attention should also be called to the results of experiment as to the best methods of securing efficiency in the ordinary number work that is met in practical life. While a great deal of this experiment has been ill advised and fruitless, a number of good results have been secured.[1] We at least seem to be coming again to recognize that efficiency in number work can not be secured without thoroughly learning the necessary number combinations and without attaining a mechanical skill that can be secured only through a large amount of computation—a belief that has not been uniformly held in recent

[1] For details see the report of Committee I.

years by our leaders in education. The sacrifice of reasonable efficiency in computation to the supposed interest of the pupil is entirely unnecessary.

Experiments.—Among the experiments that have been tried have been the basing of all number work upon measurement, and usually upon the measurement of a body of material so restricted as to be uninteresting; the extreme spiral arrangement of matter, in which the important topics, instead of being treated thoroughly once or twice, are touched upon briefly from time to time, on an increasing spiral of difficulty; making arithmetic purely technical in a narrow field, with the idea that new motive is thereby supplied and with the result that the pupil fails to acquire that appreciation of the subject that comes from a wide range of applications; the rearrangement of material at the whim of some local and temporary authority, with no real motive save a desire for change; the entire neglect of applications in the desire to attain mechanical efficiency; the neglect of abstract work in the idea that motive was to be found only in applications; the attempt to make arithmetic a mere incident to other work, using it only as a tool when a need for it arises in manual training, domestic art, or some other subject; and the attempt to accomplish the desired results by teaching arithmetic for only three or four years. Such experiments have generally been disastrous, even with the backing of enthusiasm given by their authors. Indeed, one of the most serious causes of poor work in arithmetic in the United States to-day is the feeling on the part of a few city and State authorities that they are expected to initiate their brief term of office by having a few teachers prepare a new syllabus in the various subjects of the course. It is often considered that such a syllabus is to be commended the more in so far as it resembles all the other syllabi the less.

THE QUESTION OF ALGEBRA AND GEOMETRY.

Gradual introduction.—With the abandonment of a number of obsolete topics during the past quarter of a century has come the possibility of reducing the time devoted to arithmetic, of supplying other topics of the modern business world, or of taking advantage of this saving of time by introducing a year of algebra and geometry. As a matter of fact, all three of these results have been partly attained. The time given to arithmetic at present is considerably less than it was in the old rural school, in which reading, writing, geography, and arithmetic made up most of the curriculum. Modern business problems have replaced a good many problems of the older type, such as those involving unused compound numbers, compound proportion, equation of payments, and progressions. Mensuration

in arithmetic has become a phase of concrete geometry, and the rules of the subject are no longer blindly given and blindly received. The use of the letter "x," and in general of any helpful literal notation, has found its way into the arithmetic of the last two years of the elementary school (the child's seventh and eighth school years). Thus, without any sudden revolution, all three of these changes have been or are being effected solely by the force of general opinion on the part of the great body of teachers.

Partial failures.—Numerous efforts have, however, been made to introduce algebra as such, or some form of geometry as a separate topic, into the curriculum of the elementary school. In general such efforts have not met the expectations of their advocates. Several reasons may be adduced for this partial failure. Probably the most cogent one is the fact that many teachers in the elementary school do not know algebra well enough to appreciate its value, to manipulate it skillfully, and to inspire their pupils with respect for its use. In our better class of schools this is not the case, for these schools employ well-trained teachers; but in the average school it is not unusual to find teachers who either have not studied algebra or have so forgotten the subject as to make them incapable of teaching it, even in its simplest phases. In time this state of affairs will be remedied, unless the pernicious attack of some of our authorities upon all serious mathematics in the high school shall result in the lowering of intellectual standards to such an extent as to disqualify all teachers for any thoughtful work in arithmetic.

A second reason for the partial failure of algebra may be found in the fact that our curricula have seldom been planned in such way as to give the subject much chance of success. In most cases where algebra has been introduced in the grades, it has been merely as a condensation of the formal algebra of the high school, meaningless in its content and tainted with the danger that it would lessen the pupil's interest in the subject when he reached the high school. Akin to this reason was the further one that the applied problems were not chosen with the same good judgment that appeared in the preparation of the better class of arithmetics.

It is the opinion of many teachers, however, that algebra is certain to find some place in the elementary school, at least in the form of literal arithmetic. There are two strong reasons for this opinion. In the first place, the linear equation in one unknown quantity throws light upon many problems in arithmetic, and the failure to use it for this purpose is an evidence that older and more difficult methods of solution are preferred to modern and simple ones. All inverse cases in percentage are simplified materially by the use of the letter "x" to represent the quantity sought, and it seems to be only the ignor-

ance of certain teachers that has deprived others of the opportunity of using this device.

Algebra in the eighth grade.—There is a second powerful reason for introducing literal arithmetic in the eighth grade, if not earlier. This is the growing use of the formula in artisans' journals and handbooks. This has rendered a knowledge of simple formulas essential if one would read understandingly such publications as these, and has made this knowledge of practical value if these formulas were to be used and were to lead to the development of others.

We therefore have two strong practical reasons for some algebra in the elementary grades: (1) Its use in arithmetical problems, and (2) its value in the interpretation of industrial formulas. Added to this are the pleasure and the training in exact expression that come from the study itself when intelligently presented, even with problems that have no direct bearing upon other interests, a pleasure that otherwise would never be experienced by the considerable number of those who do not enter the high school.

Introduction of geometry.—With respect to geometry the same thing is true. Efforts to introduce it as a separate topic in the eighth grade have not generally been successful. As mensuration in arithmetic it has constantly improved, but as an independent subject, whether called "observational," "intuitive," "constructive," or "concrete" geometry, it has not succeeded in the American schools as well as it should. There seems no doubt that this is largely due to the fact that so many teachers have either not studied geometry or have been assured by some authority in which they had confidence that it has no place in the curriculum. If it should become, as some would make it, either a mere elective subject in the high school or an ill-arranged set of rules of mensuration, our future teachers will be even less able to give it any status in the grades. It should be said, however, that in certain of our secondary schools where the work covers more than four years distinctly valuable results are being obtained in geometry, even in the earliest classes. In these cases the work is undertaken in somewhat the same manner as in the best European schools.

There seems to be no reasonable doubt that there will in due time appear some more systematic work in geometry for the elementary school than any that has yet been developed. It seems fairly certain that this will not be extreme in its industrial tendency, in its mensuration work, in its appeal to intuition, in its logical formalism, or in its application to design. Where such work has succeeded, it has combined these features, and if we are to have a course developed that will be a substitute for the very satisfactory work that we now have in mensuration, and that will compel the respect of our great body of earnest teachers, it will probably be of this general nature.

Objections to mathematics.—It has been intimated that certain of our educators hold an unfavorable opinion of mathematics in the high schools. Such is unfortunately the case. Whether it arises from the fact that they were unfortunate in the instruction which they received as students, or because of their blinding interests in some line of work which they are exploiting, or from some other cause, it is difficult to say. It is certain, however, that they exert an unfortunate influence upon the maintenance of intellectual ideals, and tend by their doctrines to unfit teachers in the grades for the thoughtful presentation of even arithmetic. As will be stated later in this report, the influence of such educators has not been marked.

Beginning mathematics earlier.—It naturally will occur to the foreign reader, as it continually occurs to us, that algebra and geometry, as independent subjects, might begin earlier in our schools, as is frequently the case abroad, and might cover a much longer period. But the adoption of such a plan would be impracticable under our present form of organization. It would mean that instruction which imperatively demands a teacher trained in mathematics would be intrusted to a general teacher of all the subjects, whose own available knowledge of algebra and geometry may be very limited. Whenever the plan of including in the high school the last two years of the elementary grades,[1] and departmental teaching by the present high-school teacher begins with the seventh school year, undoubtedly the algebra and geometry may safely begin earlier than at present and extend over a longer period; but in the public school as it now stands our hope for improvement in this line lies either in the gradual introduction of practical algebra into arithmetic, or in the preparation of elementary textbooks in utilitarian algebra and geometry for those who do not propose to study mathematics in the high school.

THE DANGER OF UNIFORMITY.

Early uniformity.—It has come down to us from our Declaration of Independence that "all men are created equal," and into this phrase we have read much that we knew was not true and much that has influenced our education, and in particular our elementary mathematics. For we have read into it the idea that all children have the same tastes for, need for, and ability in this and other subjects of study, and that all should take the same course with a view to having equal opportunities. So emphatic has been this opinion that only a few years ago it was commonly asserted that the course of study should be the same in all schools, and that the pupil preparing for college should follow the same curriculum as the boy

[1] See the report of Committee III, subcommittee 8.

who was going into the shop or the girl who was looking forward to domestic service or to the home.

Of late there has been a natural reaction against all of this extreme view, and as a result we are now having various types of industrial classes, apprentice schools, trade schools, and the like, the mathematical work of all of which is described in the reports of Committees II and VI.

Neglect of mathematics.—In all of these movements, however, the first thought has been given to the pupil who does not wish to learn mathematics, or at most to the apathetic one, rather than to the one who wishes to learn. Our uniformity in the past led to mediocrity, and our specializing of the present is only for the boy or girl who is looking for the purely utilitarian. That many schools have secured satisfactory results is in spite of, rather than because of, such conditions.

It would seem that a desirable step in elementary public education would be to make some provision for the pupil who wishes to progress more rapidly in one or more lines of serious study. It is possible that some plan may be evolved that will allow us, at the end of the first six school years, to differentiate our pupils, requiring all to take at least as much mathematics as at present, but allowing a select group to proceed more rapidly and to cover more ground. If this could be repeated later, we should be able to give to everyone the utilities of the subject while allowing the few to go at a more rapid pace, to do the work more thoroughly, and to cover more topics. It is too early to assert that we have reached a period in our development when such a plan is feasible, but herein we may possibly see a move that may profitably be made in the teaching of elementary mathematics. But any plan that should mean a mere elective system in the secondary school, or a diminution in the time now given to mathematics, would be disastrous to the child's mental development. At any rate some better method than the present one of giving opportunity to the one who wishes it, while preserving a good course in mathematics for all, will probably be developed.

EFFECT OF INDUSTRIALISM.

The elementary school.—The rapid growth of industry in recent years has had its effect upon the mathematics of the elementary schools, chiefly in respect to the nature of the topics and problems in the last two years (the seventh and eighth grades). The early occupations of the people of the country were agriculture and retail trade, and the topics of arithmetic were selected accordingly. At present the urban population is increasing much more rapidly than the rural, and our industries have come to be controlled by large

corporations. As a result, the agricultural problem is less in vogue, and the problem of the city and industrial type is more prominent. For the large mass of schools this is the general effect of the change that has been mentioned.

Industrial schools.—For a small number of schools the effect has been more marked. Thus we have in Massachusetts a recent development of industrial classes, the mathematics of which it is hoped will be conditioned by the chief industry of the particular locality. In several States there is an effort to have the schools in agricultural centers relate mathematics, and other topics as well, to the work of the farm. In a number of cities there are industrial schools of various types, as described in the reports upon this subject,[1] and these have all aimed to arrange the work in arithmetic so as to meet the future needs of their pupils. The natural tendency has been to make such courses narrow, and to develop only a limited vision of the work even in arithmetic. In general they do not pretend to be more than utilitarian, confessedly limiting the powers of the children to some small field. Their value lies in the motive for work that is presented by its relation to some handicraft in which the children are engaged. For a certain type of mind they are valuable, but they do not contribute to any considerable broadening of this mind, nor do they seek to add that general culture which has been found to make for happiness and for sympathetic interest in the world at large in later life. They tend to close the door to general mathematics, and they are to this field what a course in the literature of artisans' manuals would be to the nobler literature of the language. It must, however, be recognized that there exists a type of mind to which this work appeals, and that for this type such work is necessary. In so far as the school is right in its judgment that a pupil is not worthy of something better, such a narrow range of work is to be commended. It is to be hoped that courses will be organized that may worthily meet this condition; but it is also to be hoped that the schools will recognize that the well-trained all-round citizen needs a more general education than this type of work in mathematics suggests.

Practical problems.—Along with this movement there has proceeded an effort to add motive to elementary mathematics (almost exclusively arithmetic) by relating the problems solely to the manual arts and to the purchases and other immediate needs of the children from grade to grade. Within reasonable limits problems of this type are valuable, but it is easily seen that they may become worse than the oldest types that we have inherited. A problem about bookbinding is of no more value than one about the price of a cow unless

[1] See the reports of Committees II and VI.

the pupil is binding a book; indeed, it is probably distinctly less useful, since the value of a cow has the more important information. Unless, therefore, special manual-training problems are directly correlated with the corresponding manual work, they are usually worse than the conventional general type that has stood the test of time, provided always that this type is modern in its content. For this reason there has not, in this country, been any particular interest shown by the general type of school, either in the problems of some narrow field of industry or in those that relate to the industrial and domestic arts, except as introduced directly in classes in those subjects. It has generally been found that there should be a good all-round course in arithmetic, with modern problems of a general type, and that it should be left to the classes in the various special arts to draw upon the knowledge thus secured to solve their own particular problems.

THE TEACHERS.

In general the teachers in the elementary schools in the United States are women, the reasons for which have been set forth in the report of Committee I and elsewhere in this report. In some cases men are employed in the higher classes. These women teachers are, in the cities and large villages, generally graduates of normal schools, of city training schools, or of high schools. They have to pass a State or a city examination for a license to teach, and they form an efficient and devoted body of workers. In the rural schools and in the smaller villages there is need for improvement, owing to the peculiar conditions in which our country finds itself at present. Many teachers in these schools recognize, quite as well as others do, their deficiencies in education and in professional training. It can only be said that, under the conditions in which a new country finds itself, it is impossible to secure the number of trained teachers necessary to do the work as the country would like to see it done. The improvement in this direction during the past generation has, however, been gratifying. It shows itself in a greater public appreciation of the need for well-qualified instructors and in a general increased interest in the work of the school.

THE OUTLOOK FOR IMPROVEMENT.

Present criticisms.—The work in mathematics in our elementary schools has been criticized—has always been criticized—as unsatisfactory. In general it is said, and with some truth, that pupils leaving the school at the age of 14 can not add, subtract, multiply, and divide, in the domains of integers, simple common fractions and decimal fractions, with fair rapidity and with accuracy, and

that they are deficient in the ability to find per cents of numbers. The general public, and, indeed, the countinghouse, the bank, and industry, give voice to few other criticisms of arithmetic, but there is no uncertainty as to these that have been mentioned. While the general public does not profess to judge whether a child has been well taught with respect to square root, proportion, or inverse problems in percentage, and professes no great interest in the deficiency of children in matters of taxes, insurance, and investments, it does judge, and it judges harshly, this inability of children to handle the fundamental operations with certainty and with a fair degree of skill. It states with some emphasis that eight years ought to be long enough to accomplish the little that it demands, and it is probably justified in this complaint. The school is recognizing this fact and has begun to adjust its work accordingly. Recent courses of study have substituted more work in the fundamental operations for certain of the obsolete applications of arithmetic, it being the feeling that if the pupil knows the processes thoroughly, together with the common rules of mensuration, he will see how to solve the problems of his vocation when the time arrives.

Improvements suggested.—The first improvement, therefore, in the immediate future seems to be to emphasize the work in the fundamentals of arithmetic, including the domains of integers, common fractions, and decimal fractions (with percentage), using complicated cases only sparingly and at the end of the course. That this can be done without a large amount of drill work in pure number is not the general opinion of experienced teachers.

As to the methods of presenting the simple facts of number, our schools seem to be accomplishing their task in a satisfactory manner. There has been a healthy reaction against the use of narrow methods; and although individual teachers will naturally have their favorite devices (as of going to some extreme in measuring, the exploiting of the ratio idea, and the use of such a limited range of objects as inch cubes), there is a general recognition of the fact that the problem is really a simple one. At present the teacher commonly employs convenient objects, without confining herself to a narrow list, for the purpose of making number concrete and of having the number fact understood, abandoning them when this purpose has been accomplished. There is a decided tendency to recognize that number processes must become mechanical if they are to be used successfully, and this has resulted in a return to drill work on the operations, a feature that has never been abandoned with satisfactory results.

As to the applications of arithmetic, there has been a general tendency to make these appeal to the pupil's activities and interests as far as possible. There are certain large fields of application that appeal to all, relating to commodities found everywhere, to general

business customs, and to the common occupations of our people, and these find their places in the general textbook for all schools. It has been found, however, that the interests in one part of the country are not those in another part, and the outlook for improvement at present is not so marked in the way of textbook problems as in the awakening of the school to the advantages of problems of a local character. This has shown itself in the preparation of lists of problems derived from the interests of the particular locality, and occasionally in the printing of brief collections of this character.

The curriculum.—As to the arrangement of the curriculum, the outlook seems to be for a return to a plan in which a subject, such as percentage, is taught consecutively until it is sufficiently mastered, or with not more than two breaks in the sequence, as opposed to an extreme spiral method that was tried a few years ago. Under adequate supervision the plan of breaking such a topic into two or three parts, properly graded as to difficulty, has succeeded; but without this supervision, or in schools not equipped with trained teachers, the topical arrangement of matter has been more satisfactory. The outlook is that both types will be recognized, and that little by little a mean will be found between the two.

III. SECONDARY SCHOOLS.

ORGANIZATION.

General and technical secondary schools.—The secondary schools of the United States may be classified as general and technical, the former having general culture as their primary aim, while the latter aim to prepare more or less directly and completely for certain occupations. As the technical schools have arisen largely during the past decade only, and are of the most varied character, their diverse curricula are as yet in the earliest stages of their evolution, and the problem of the modification of the work in mathematics as found in the general schools, so as to adapt it more effectively to the purposes of the various types of technical schools, is one that now calls for careful study. [illegible] to the present time the conditions and tendencies in the w[illegible]thematics in the two types of schools have not different[illegible]selves sufficiently sharply to require separate discussion [illegible] point.[1]

Publ[illegible]d private schools.—Both the general and the technical schools [illegible]ay be again subdivided into public and private schools. For the general schools, the public and the private schools have been separately studied.[2] The results have shown surprising uniformity.

[1] Separate discussions are given in the reports of Committees III, IV, and VI, and their subcommittees.

[2] By Committees III and IV.

Both private and public schools have long existed in large numbers in the United States, and it might be expected that the absolute freedom on the one hand and the more or less considerable measure of central administration in the various States on the other would lead to marked difference in the work of the institutions. Yet, if direct mention of the type of school and the statistical matter pertaining directly to the type were eliminated from the report on the public general secondary schools and that on the private general secondary schools, they might well pass as two descriptions of the same class of schools written by two different committees. The agreements would be many and important, while discrepancies would be few and usually explainable by the naturally different point of view of two different committees. It is apparent from the reports themselves, and is also a matter of current knowledge among those well acquainted with the conditions in the United States, that the differences in mathematical work between the two classes as wholes is decidedly less marked than the variations within each class.

In the present general discussion we accordingly speak of secondary schools as a whole, distinguishing if need be between the good and the poor, the progressive and the ultra-conservative, but seldom between the public and the private, the general and the technical.

Admission to the secondary school.—The secondary school receives pupils on examination or on certificate after completion of the elementary school course. The normal age of admission is 14 years, and the course of study usually covers 4 years.

THE CURRICULUM.

Recent changes in the curriculum in mathematics.—The last few decades have witnessed no thoroughgoing remodeling throughout the United States of the secondary curriculum in mathematics at all comparable with those that have taken place in several European countries. What changes have taken place have been made by a relatively small portion of the schools acting singly, have varied widely according to local conditions, and have left the situation as a whole but little changed.

One must not, however, make the mistake of thinking that America is complacently folding her hands in the belief that in the matter of mathematical curriculum she has reached the acme of perfection. This is far from being the case, but freedom from central control and supervision has been accompanied by absence of central stimulation and leadership, and the distribution of ultimate authority among thousands of local centers has had as perhaps inevitable consequence a condition of affairs in which few of these centers feel themselves strong enough to initiate and carry through any important change.

For while the course of study in mathematics is nominally fixed by the governing body of the school, after conference with its principal, and to some extent with the teachers of mathematics, it is in reality determined almost entirely by the entrance requirements of the colleges.[1] The schools, acting singly, seldom venture to make changes in their work which would throw it out of conformity with the current entrance requirements of the colleges to which their pupils usually go. The colleges, on the other hand, must consider the capabilities of the schools in fixing these requirements. The accomplishment of the schools and the requirements of the colleges being at present in working equilibrium, any one school or college introducing a radical change single handed might find itself not so much initiating a general reform as diverting students from itself. Under our present custom of action (or inaction) by single institutions, not all institutions feel either the responsibility or the ability singly to grapple with national problems, but all feel keenly the strong pressure holding each single institution to conformity to present standards and customs.

In this connection an important opportunity of leadership is offered the strong colleges, particularly those of private foundation. If the State university, as part of the general State educational organization, is obliged to accept students as prepared by the generality of the schools of the State, the privately endowed institution is free to make use of entrance requirements to select students whose preparation has been satisfactory, and on this basis to give them a better education than would otherwise be possible. The experience of leading colleges acting singly in past decades shows that this is not an impossible ideal. Under concerted study and action, the outlook would of course be decidedly more promising.

The curriculum in mathematics.—The curriculum in mathematics in secondary institutions with a full course of four years varies but little in the great majority of cases from the following average (the arabic numerals denote the work of half a year, 18 to 20 weeks, 4 or 5 periods of 40 to 50 minutes each, weekly):

FIRST SCHOOL YEAR.

1; 2. *First course in algebra*, including literal notation, the four fundamental operations for positive and negative numbers, fractions, factoring, linear equations in one or more unknowns, problems leading to such equations, quadratic equations with numerical coefficients and simple problems leading to such equations. There is a tendency to use problems related to the life of the day, when available. Graphic methods are used to some extent.[2]

[1] See report of Committee III, p. 15.

[2] For a more detailed account of the work in the various subjects, see the report of Committee III, p. 17 et seq.

SECOND SCHOOL YEAR.

3; 4. *Plane geometry.* This course covers five "books": I. Rectilinear figures; II. The circle; III. Similar polygons; IV. Areas of polygons; V. Regular polygons and the measurement of the circle. The work includes the more or less "original" demonstration of a goodly number of easy or moderately difficult exercises, including problems in loci, and constitutes all the formal instruction in elementary plane geometry that the pupil ever receives, in school, college, or university.

THIRD SCHOOL YEAR.

5. *Second course in algebra.* Radicals and exponents, but not logarithms. Quadratic equations in one unknown: solution and problems, with some attention to the discriminant and the relation between the roots and the coefficients. A few higher equations solved by means of quadratics, and a few special cases of systems of equations with two unknowns solved by means of quadratics. The elements of arithmetic and geometric progressions. Tendencies as under 1 and 2.

6. *Solid geometry.* This covers four "books": VI. Lines and planes in space; VII. Prisms and pyramids; VIII. Cylinders and cones; IX. The sphere. The subjects are treated as under 3. 4. and constitute the totality of the formal instruction in elementary solid geometry given in school, college, or university.

FOURTH SCHOOL YEAR.

7. *Elementary plane trigonometry.*
8. *Third course in algebra.*

Courses 1–4 are generally required of all pupils and course 5 is frequently so required, while courses 6–8 are usually elective. A course in commercial arithmetic is also sometimes offered. As course 5 is always required for admission to colleges and technological schools, while course 6 is generally so required by technological schools and sometimes by colleges, these subjects are usually given by the schools and are elected by a considerable number of pupils.

Course 7 and course 8 are given by practically all colleges, and in practice are regarded as primarily collegiate work. They are accordingly given by relatively few schools and elected by relatively few pupils in those schools. The ground covered and the method of the treatment are largely influenced by the wish of the school and the pupil, that the work done may be accepted by the college to which the pupil goes.

The standard secondary-school course in mathematics as it actually exists to-day may be regarded as consisting of courses 1–6, described above. There are some variations in order, but the above may fairly be regarded as typical. As a textbook is always used, and as the teaching usually adheres closely to the text, a good idea of the details of the ground covered may be obtained from the examination of some of the books in current use.

Tendencies to modification.—Marked tendencies to change the curriculum in various details are distinctly noticeable in the country, and seem to be gaining in strength. Thus, there are tendencies to omit

geometric proofs that are either obvious or too difficult; to transfer the more difficult portions of the algebraic matter hitherto given in the first year of the high school to a later year; to avoid algebraic manipulations of greater complexity than is requisite to prepare pupils thoroughly for the work that lies beyond; to give more prominence to the equation; and to introduce more problems from physics and other sciences and from practical life.

Simultaneous teaching of algebra and geometry.—It has been proposed to redistribute the subject matter of algebra and geometry as now taught in the secondary schools (without altering either the ground ultimately covered or the total amount of the time given to mathematics) so that algebra and geometry should be taught simultaneously during the years in which they are now taught successively. In considering such a rearrangement there arises first of all the question of its theoretic desirability; should this be decided affirmatively, that of its feasibility presents itself. From the point of view of educational theory it may be urged that the simultaneous teaching of algebra and geometry is desirable, because it permits the gradual development of both subjects during a long period, thus facilitating digestion and assimilation; because it allows the postponement of the more abstract and difficult portions of each subject to the later years of the curriculum when the pupil is more mature, both mathematically and in general; because it makes possible the use of the methods and results of either subject whenever advantageous in the treatment of problems of the other; and because it eliminates the detrimental periods of disuse of either one of these subjects or of all mathematics.[1]

On the other hand, it may be urged that the simultaneous teaching of algebra and geometry is undesirable, because many teachers fear that at the age of 14 the pupil's mind is not sufficiently mature to begin demonstrative geometry without distinctly lowering the standard of the instruction, a result that would be decidedly deplorable; because the points of direct contact between algebra and geometry are so few that the simultaneous treatment allows for little if any more correlation than the tandem arrangement; because there is already a great deal of practical mensuration that has been included in arithmetic on which to draw in the first treatment of algebra; because there is also included in arithmetic the general idea of similar figures in connection with the applications of proportion; and correlation with the work in areas, solids, and similar figures is therefore possible in the first course in algebra at present and is actually effected by a large number of teachers; and because when geometry is reached correlation with the algebra that has preceded is feasible under the present order.

[1] See report of Committee VII, pp. 20, 21.

Considering the practical aspect of the plan, those who advocate the simultaneous teaching of algebra and geometry point to the fact that it has been long and thoroughly tried by some of the leading nations of the world; that these nations have found simultaneous teaching entirely feasible and have achieved excellent results under it; and that every one of them is convinced that the simultaneous teaching of these subjects is decidedly preferable to their successive teaching.

It does not follow, of course, that the plan must needs be practicable under American conditions. In determining whether or not this is the case, the measure of general preparation of our teachers, their degree of readiness to handle this particular plan, the heterogeneous character of our pupils, the number of pupils that transfer from one city to another in the midst of their secondary course, the existence of suitable texts, and the temporary disturbance occasioned by the change must all be taken into account.

The plan has actually been tried in a few American institutions with divergent results. In some cases the experiment has proved satisfactory and the use of the plan is continued;[1] in others it has been tried and abandoned, partly because teachers have found that by teaching geometry only two days in the week the interest of the pupil is not as easily carried over from one class exercise to another as when the subject is taken up every day.

It must not be understood that the simultaneous teaching of algebra and geometry means an equal distribution of the weekly time between the two subjects. Such a mechanical attempt at simultaneous teaching would offer the minimum promise of success. Simultaneous teaching as used above means rather a "block system," a distribution of the subject matter so that in each year blocks of algebra and geometry alternate, each block having a measure of unity and completeness in itself. The present curriculum for the first two years may be described as a system of two blocks. Those who may be disposed to try simultaneous teaching would find a system of four blocks for these years a conservative and advisable beginning. If this should prove successful, the blocks may gradually be made smaller as experience dictates.

Nor should simultaneous teaching be understood to imply a "fusion" of the two subjects into a homogeneous whole which is neither one subject nor the other. Such a fusion is impossible, owing to the inherently diverse characters of the two subjects. It would in any event be undesirable, and attempts at approximation to it are not to be commended.

[1] Report of Committee IV, pp. 150–159.

THE INSTRUCTION IN MATHEMATICS.

Aims.—In the nontechnical secondary schools the teaching of mathematics is regarded as having for its chief aims the mental development of the pupil and his preparation for later scientific work.[1] It is held that the essential benefits of the study of mathematics can be attained as readily and as fully in the study of the subject matter required for admission to college as in any other selection of topics, and consequently the courses in mathematics offered to all pupils conform to the current collegiate entrance requirements.

Two tendencies in the thought of the country concerning the aims of teaching secondary mathematics should be mentioned—a tendency to modify the conception of the aim of the teaching to conform to what is understood to be the outcome of recent psychologic research concerning the value of "formal discipline," and a tendency to attach greater importance to the utilitarian possibilities of mathematics.

With respect to the former, workers in mathematics have every reason to be gratified by the progress that is making in the utilization of the mathematical type of thought in the treatment of the problems of psychology and in other complex fields; in the predominance that precise thinking is gaining over vague theorizing in so many domains of the scientific and practical activities of the day they will see added reason for the careful training of the coming generation in habits of precise thinking, beginning with that simple and readily mastered domain, commonly known as elementary mathematics, which has been and is the inspiration and model of precise thinking in more difficult fields.

That mathematicians will accept and take to heart the results of scientific investigations that bear upon their science or its teaching goes without saying. In fact there is danger that, in unhesitating confidence in the dominance of the mathematical idea in modern scientific thought, they will too readily accept as established with a definiteness akin to that of mathematics results announced by students of a sister science without ascertaining that the conditions really warrant such acceptance.

With respect to the so-called "doctrine of formal discipline" in particular there is danger that results of psychologic research may be misunderstood and misapplied by one who accepts it as a settled fact that these researches have "exploded the doctrine of formal discipline as generally stated," without any statement of what the doctrine is or how it is "exploded." The mathematical thinker

[1] It is instructive to compare with these the aims set forth for the study of graduate mathematics in the report of Committee XII, p. 12, and cited in this report on p. 48.

will demand that the "doctrine" in question be explicitly and clearly defined, and that the experiments and their results be stated with equal definiteness. He will then ask himself in how far he has actually held the view that is stated; he will consider the number and character of the experiments and the personal equation of the investigator, and will determine what it is that the experimental results really prove; and finally he will decide what modifications, if any, these results require him to make in his attitude. At the present time few, if any, mathematicians who are conversant with the results of such experiments as have been made, and are sympathetic with their spirit, feel that aught has been as yet established which would require them to change their views of the value of the study of mathematics.

The mathematician welcomes every effort to substitute careful experiment and exact results for loose thinking and dogmatic generalizations, and in this connection recent work on measuring the mathematical ability of children and systematically testing their mathematical progress may be mentioned as constituting a suggestive beginning in a large and important field.[1]

As regards the second tendency, that of the increased emphasis on the utilitarian values of mathematics, it may be noted that this tendency, praiseworthy as it is, carries with it an element of danger. The practical aspects of mathematics deserve attention not only on account of their inherent value, but also on account of the additional interest which they may arouse; and the effort to exhibit the rôle that mathematics plays in the world about us deserves only commendation; the danger connected with this tendency lies not in the introduction of utilitarian applications, but in the attitude of mind which the search for such applications may engender. The very variety and importance of the uses of mathematics in trade and industry may tend to foster the opinion that the learning of such uses is the chief end of the teaching of mathematics. This is, of course, no more the case with respect to mathematics than it is with respect to less utilitarian subjects of the curriculum. History and English literature are taught not because they can be directly used in earning dollars, but because the educated person should know something of these fields and because the mind of the pupil will grow along desirable lines through thinking their types of thought. Mathematics is taught for quite similar reasons. Its utilitarian possibilities should not cause us to forget that the main purposes of its teaching are to acquaint the pupil with the content of a portion of a domain of thought that is fundamentally characteristic of the human mind as such, and not of any particular time, place, or civilization, and to

[1] See report of Committee I, pp. 86, 91.

cause his mind to grow along desirable lines by actually thinking in the type of thought that is characteristic of this domain.

Mode of instruction.—The form of instruction varies widely with different teachers, to whom large latitude is usually allowed in this respect. Only in case of a very unusual plan would it perhaps be necessary to seek the permission of a superior officer (say the principal), and a serious teacher of good standing would have no difficulty in securing it. Its renewal would of course depend on the degree of success attained on the first trial.

As practically constant features of instruction there may be named: The use of a text in the hands of the pupils, containing both theory and exercises, to which the instruction conforms closely; preparation at home of assignments from the text, usually with preparatory explanations in the class; many blackboards (as a rule occupying nearly or quite all the available wall space of the room), on which the pupils can write singly or simultaneously the solutions of the problems worked out at home or solve others extemporaneously. Some competent teachers successfully develop the subject matter of geometry with the class without the use of a text.[1]

PREPARATION OF TEACHERS.

The growth of the facilities for preparation.—The systematic preparation of teachers of secondary mathematics in the United States is but in its infancy. In the collegiate mathematical instruction of two decades ago practically no special attention was paid to the needs of prospective teachers of secondary mathematics. The opportunity to make acquaintance with mathematical subject matter in advance of that taught in the high school had indeed long been offered by the colleges, of which even the smallest usually gave instruction in mathematics sufficiently advanced to include at least a course in plane analytic geometry and a fairly extensive first course in the calculus. But the instruction in these courses took no account of the possible future occupations of the students, and it may well be doubted whether the thought that some member of the class might later teach mathematics often occurred to the collegiate instructor with sufficient force to influence his work even slightly. No separate institutions for the training of secondary teachers existed.

Nearly two decades ago,[2] however, there began to be given courses of collegiate grade considering the subject matter of elementary mathematics and the methods of teaching it with reference to the needs of teachers of secondary mathematics. The number of institutions giving such courses has increased rapidly and is still growing

[1] For details as to modes of instruction see the reports of Committees III and IV.
[2] See report of Committee V, pp. 5–8.

to-day.[1] Further, there is every reason to believe that instructors of collegiate mathematics are increasingly conscious of the presence of future teachers of secondary mathematics in their classes, and that they are more and more frequently availing themselves of such opportunities as arise to point out the bearing of the work in hand on elementary mathematics.

There have also been founded in the past two decades various separate institutions of collegiate rank or independent divisions of universities having the training of secondary teachers as one of their chief functions.[2] These institutions all have departments devoted especially to mathematics; their number is still growing; the stronger ones have already become potent influences in the teaching of mathematics in their environment and in the country at large, and they give every promise of becoming still more effective agencies of progress in the years to come.

It is especially gratifying to note that the institutions which in one form or another are giving courses on the teaching of secondary mathematics are, as a rule, well equipped to give adequate instruction in a range of mathematical subject matter sufficiently wide to meet the needs of the prospective teacher, and thus are prepared to base a sane study of the problems of teaching upon a sound foundation of mathematical knowledge. In this way there is obviated the not altogether unwarranted distrust of an elaborate study of "methods" based upon a meager fund of "matter."

The past two decades may be said to have witnessed the beginning of the systematic training of secondary teachers in this country and its development to a sufficient extent to mark the probable lines of its further growth. Corresponding provisions for the needs of the elementary teacher had been made earlier by the formation of the normal-school system, and that for the teacher of collegiate mathematics may be regarded as a problem of the immediate future. The practice that is occasionally found of giving advanced graduate students the privilege of serving in early collegiate courses as assistants to experienced instructors and of teaching under their direction and supervision is a considerable contribution to the solution of this problem. Such work should, however, be distinctly a course of study and not service rendered for remuneration. The practice now so prevalent among advanced students of diverting their energies from scientific work to the instruction of classes for the purpose of earning funds to meet current expenses is a serious evil.[3]

It is also possible that efforts to make provision for the needs of the prospective teacher of collegiate mathematics will take the shape

[1] See report of Committee III, pp. 79–80.
[2] See report of Committee V, p. 7.
[3] See also this report, p. 56.

of an enlargement of the scope of the work of the stronger institutions that are already covering the secondary field.

The present degree of preparation.—At present the strong newly appointed teacher of mathematics is a college graduate who has had a year's course in the calculus, with antecedent courses, making a total of from 180 to 360 class periods of collegiate work in mathematics. Usually he has had varying additional amounts of mathematical work, and occasionally he has had some training in the theory and practice of teaching the subject.

The average newly appointed teacher of mathematics is a college graduate[1] who has had only about one year's work (from 90 to 180 class hours) of mathematics beyond the work of the school in which he teaches.[2]

The constitution of this year's work varies somewhat, but a typical combination would be: Trigonometry, college algebra, analytic geometry. The average preparation includes no strictly professional training, no course in the teaching of mathematics to initiate the candidate into the teacher's mode of viewing the events of the classroom. Consequently, he enters upon his work with but little mathematical knowledge in advance of his pupils, and with no training at all in the technique of the work he is about to undertake. He is essentially a former pupil, somewhat matured by the general experiences of his college studies and life, come back to teach his quondam fellows.

If successful in his work, he develops into a good teacher, at the expense of many mistakes, more or less numerous and serious, according to his measure of native aptitude for the work of a teacher. In exceptional cases he extends his mathematical attainments further by means of attendance on summer sessions of higher institutions, by private study, or by correspondence work.[3] But it undoubtedly requires special enthusiasm and devotion to do this in addition to the heavy burden of teaching work generally carried.

It is obvious that this measure of preparation is entirely inadequate and that, while pioneer conditions may have required the toleration of such a state of affairs up to the present, the time is now at hand for the adoption of a radically different course.

In this connection the urgent need of taking effective steps to make the work of teaching secondary mathematics more attractive to men should also be mentioned. While the devoted service given by women in the mathematical work of secondary schools and the earnest efforts they are putting forth to meet every educational requirement deserve only warmest commendation, and while there are

[1] In the totality of teachers already in service the proportion of college graduates varies largely. See report of Committee III, pp. 76–78.

[2] See report of Committee III, pp. 76–77.

[3] See report of Committee VIII.

many women whose mathematical aptitudes and proclivities fit them to reach the highest level of efficiency in secondary work, it is undoubtedly true that on the whole the intellectual interests of women do not lie in pure mathematics to the same extent as do those of men, and still less is this the case in the applications of mathematics to physics and other sciences or to engineering and other occupations. Mathematics is thus a field which furnishes exceptional opportunity for impressing on the pupil the man's point of view and his way of grappling with difficulties, a feature of instruction which should be prominent in every well-rounded scheme of secondary education.

Tendency to raise the standard.—A marked tendency toward raising the standard of new appointees is to-day to be found throughout the country. This is due in part to the awakening of a keener consciousness of the desirability of higher standards and in part to the greater supply of candidates who have had some collegiate or even graduate mathematical and pedagogic training. In consequence of the work done in the departments of mathematics in the graduate schools, in the more advanced undergraduate work of many colleges, and in the professional schools for the preparation of secondary teachers, large and increasing numbers of candidates whose training is decidedly beyond the average stated above are constantly sent forth. Those who have attained the doctor's degree in mathematics have no difficulty in securing collegiate positions, but those who have only advanced part way through the graduate school or who have merely attained the bachelor's degree, even though specializing in mathematics, have little ground to expect to secure a desirable collegiate appointment. These candidates, as well as those prepared by the professional schools, must usually find their posts in the secondary field. Thus the average attainments of the new appointee are gradually and surely being raised by the mechanical action of natural causes. But more rapid amelioration of conditions is desirable than will be achieved under the slow and blind action of natural conditions, and earnest efforts are requisite to accelerate the rate of progress, for undoubtedly the greatest single need of the day in the field of secondary mathematics is the better preparation of teachers. Any material advance in this respect would have salutary effects far beyond the domain of actual instruction. A body of adequately equipped teachers would not only insure adequate treatment of the problems within their control, but would also stimulate decidedly better treatment of those not directly subject to their control.

From the practical point of view, however, it must be noted that even under the present standards the supply of well-prepared teachers of mathematics falls short of the demand, especially in the case of men, and it might seem futile to raise the standard still higher.

In reply, it may be asked whether all the schools that are now requiring better preparation have correspondingly advanced the remuneration offered. Are schools endeavoring to secure a much better grade of teacher for a stipend that was already too meager? Salaries proportionate to the requirements will induce candidates to prepare themselves to conform to any standard, however high; but in a country of opportunity and promise like ours, and at a time when the high cost of living is as serious as at present, salaries barely equal to the wages of skilled artisans or high-grade stenographers can hardly be expected to attract ambitious and capable young men and women into years of earnest and costly preparation for a serious and exacting occupation. Under present conditions the result will inevitably be a supply inadequate both in number and in the quality of those who formally comply with the requirements. The remedy lies almost wholly in the hands of the appointing authorities. When it is once known that high-grade pay can be obtained by high-grade candidates with high-grade preparation and by no others, the supply of such candidates will rapidly increase.

Proposed standards.—Schools that desire to appoint adequately prepared teachers look to institutions of collegiate rank not only to furnish the preparation and to bear testimony concerning the quantity and quality of the work done by the particular candidates under consideration, but also to determine what constitutes adequate preparation. Hitherto a few individual writers have drawn up lists of courses intended to outline such a minimum, and a few colleges have independently defined the attainments that a student must have to receive the collegiate recommendation.

These lists are meant to indicate the selection from the work now offered by good colleges that may properly form the minimum requirement for all candidates at the present time. The lists are in substantial agreement and include, in addition to the subjects taught in the schools, good courses in trigonometry, college algebra, analytic geometry, the differential and integral calculus, determinants and the theory of equations, modern geometry, the elements of analytic mechanics, and at least a little work in theoretical and practical physics. Such courses on the history and the teaching of secondary mathematics as are offered by the college should also be taken.

With a little enrichment of collegiate instruction,[1] it would be possible to require the following minimum:

(*a*) Trigonometry, college algebra, analytic geometry.
(*b*) Surveying, or descriptive geometry, or elementary astronomy.
(*c*) The differential and integral calculus with applications to geometry, mechanics, and physics.
(*d*) Modern geometry.

[1] See the discussion of advanced collegiate electives, p. 45 of this report.

(e) The elements of analytic mechanics.
(f) The elements of theoretic and laboratory physics.
(g) Algebra from a modern standpoint.
(h) One or more courses introductory to important fields of modern mathematics.
(i) One or more courses in the history of mathematics.[1]
(j) One or more courses on the teaching of mathematics.[2]

It would be desirable if the best secondary positions were attainable only by candidates who have had, in addition to the subjects (a) to (j), at least a year's work in graduate mathematics culminating in the master's degree, and this measure of attainment is already required in some schools.

We may repeat that secondary institutions can secure candidates of any desired degree of thoroughness of preparation by offering sufficiently attractive inducements in the form of remuneration and conditions of work. When once they let it be known that such inducements are permanently offered, an ample supply of desirable candidates will soon be forthcoming.

Progress by teachers in service.—Within the past score of years there have grown up important facilities for teachers in service to make progress in mathematics without resigning their positions—notably the summer sessions of a goodly number of colleges and universities[3] and the various organizations of teachers that have been formed. These organizations have done effective and most desirable work in arousing interest, in bringing secondary and collegiate instructors of mathematics into closer relations, and also in contributing largely to such improvements of detail in secondary mathematical work as have taken place since their formation,[4] although they have not as yet led to any noteworthy general reform. They have no doubt stimulated many teachers to individual thought and progress, and they have within their reach the possibility of participating valuably and with ever-increasing effectiveness in the mathematical growth of the times.

The secondary teacher and the textbook.—Under present conditions as to the preparation of teachers, it is inevitable that the text should be regarded as a fairly complete record of all needful matter and the ultimate standard of instruction. Geometry and algebra usually mean to the beginning teacher but little more than the body of material in the books from which he was taught. In this belief

[1] These courses may cover contemporary as well as past conditions and should utilize the vast amount of valuable material furnished by the reports to the International Commission on the Teaching of Mathematics. Messrs. Georg & Co., Geneva, Switzerland, are the selling agents for all these reports.

[2] These courses should include critical discussion of the subject matter, considerable observation of instruction, and whenever possible practice teaching.

[3] See report of Committee VIII, p. 86.

[4] See report of Committee III, pp. 15, 24.

he and his teachers to the n^{th} generation have been taught, and in this belief he teaches, except in the rare instances in which the college has effectively disturbed the tradition. It might seem that the textbook could readily be made a potent instrument of improvement, but there are serious obstacles in the way of the realization of this hope. The text is selected by the teacher, or for the teacher, in view of his abilities and limitations. Texts with marked deviations from those in current use would seem to many teachers no longer really "algebras" or "geometries." Such deviations would undoubtedly burden the teacher with the task of learning afresh whatever is new in the book. Even a change from one text to another along the same lines brings new sets of exercises for classroom use. Consequently the average teacher is conservative in the matter of textbook changes. Reform through textbooks alone would at best be feasible only to a restricted extent, because the text must be adapted to the pupil, while the progress which the teacher must make to improve his work can be aided chiefly by material in nowise adapted to the mind of the pupil.

Furthermore, publishing houses[1] take pains to secure the opinion of the average teacher and of those who know him, and a manuscript that does not bid fair to win the approval of the teaching body in sufficient measure to secure a remunerative sale would not easily secure an effective publisher. The influences at work tend toward the preservation of the present status rather than toward its amelioration. This is strikingly seen in the remarkable similarity of the current secondary texts in mathematics, and in the almost complete absence in the United States of a highly diversified body of literature relative to the field of secondary mathematics, such as is found in some other countries, made up of works, good and poor, varying widely and in many different directions from current usage, sometimes erratic, but often stimulating thought and leading the way to better things. Our publishing houses are not philanthropic institutions and can not be expected to publish books that give every promise of loss; but there is room for a valuable piece of philanthropic work in establishing some agency for the publication of good mathematical works concerning which competent critics would render not the verdict, "The average teacher will probably like this book and adopt it as text," but the verdict, "The good teacher ought to like and use this book."

CONCLUSION.

On general survey of the secondary field two main needs strikingly dominate the whole mathematical situation: The need for the better preparation of teachers and the need to reduce, if not elimi-

[1] See report of Committee VIII.

nate, the waste of effort involved in independent and often inadequate treatment of fundamental and broad questions by separate schools, colleges, or local systems.

The first of these needs must be met by gradual development; perhaps all that can be done by individuals is that each should take special pains to stimulate progress on this line whenever and however possible. It will not suffice merely to raise the requirements for appointment; there must be an accompanying guaranty of adequate remuneration and suitable conditions of work. To secure this guaranty is mainly an administrative problem, often a political one, and must at present be dealt with as may be possible through these channels.

As to the second, the question is well worth considering whether some form of concerted study can not be found that will succeed in focusing the best thought of the country upon those of the problems relative to the field of secondary mathematics that are really of a general character, so as to secure the preparation of recommendations that would have a moral authority commanding the nation's confidence and insuring their widespread acceptance.

IV. MATHEMATICS IN THE HIGHER INSTITUTIONS OF LEARNING.[1]

THE ELEMENTARY MATHEMATICS OF THE COLLEGE AND THE TECHNOLOGICAL SCHOOL.

The conditions that prevailed only three decades ago in the teaching of college mathematics are in marked contrast with those of the present time.[2] The typical college teacher of that period, like many school-teachers of to-day, knew little more of his subject than he was obliged to teach. In the technical schools the situation was the same. The calculus was taught like formal algebra, not only without regard to its applications outside the domain of pure mathematics, but without understanding of its nature, its principles, and its use within that domain. Little wonder that the engineers whose student days lay in those years have made relatively little use of the calculus in their profession. The causes that led to a complete change in these conditions are traced in the report of the Committee on Graduate Work,[3] and will be referred to later.

[1] For the benefit of the foreign reader we recall at this point that the higher institutions of learning in the United States consist of the colleges and the technological schools, and the universities. While the latter term is applied in certain parts of the country to institutions below the rank of a good college, it also embraces those which correspond to European standards, and in which the advanced instruction in science and letters is carried on. It is in the latter sense that the term *university* will be used in what follows.

The terms of admission to the colleges and the technological schools are fairly uniform, and so far as they relate to mathematics are considered below. The average age of entrance is between 18 and 19.

[2] These conditions are well set forth in the report of Committee XII, pp. 43, 44.

[3] Committee XII, pp. 7–10.

Entrance requirements.—The requirements in mathematics with which all pupils who are to be admitted to the better colleges and technological schools of the country are to-day obliged to conform are elementary algebra through quadratic equations, plane geometry, and sometimes solid geometry.[1] Thus the college or the technological school is enabled to arrange its program so that the student in the freshman year takes logarithms, trigonometry, and analytic geometry, although often solid geometry is also one of the studies of that year. In some colleges much time is given to algebra.[2] This is doubtless due in the case of the weaker institutions to the fact that the freshman can not solve a quadratic equation; and to the extent to which there is a tendency in school instruction in algebra to lay excessive stress on the solution of problems, and to reduce the training in pure algebra to a minimum, freshmen in all institutions are still less likely to be able to use their algebra in college work.[3] But the ultimate responsibility for such a state of affairs rests with the college, which through not having enforced its entrance requirement in algebra finds itself in the position of being obliged to give school instruction. The presence of these elementary courses in algebra in the college program is, however, also explained in part by the persistence of the formal calculus of two or three decades ago, when a course in the calculus amounted to little more than a new variation in the formalism of algebra. On the other hand, those parts of algebra that properly belong in a college course—determinants, the solution of higher equations, and various minor topics—are being taught at the present time in the better institutions in connection with their applications or with kindred subjects in analytic geometry and the calculus.

Arrangement of the material in the elementary college courses.—Some institutions have found it convenient to rearrange in a measure the topics treated in the elementary courses, and the calculus is now sometimes begun in the freshman year, the more difficult parts of analytic geometry being postponed to the following year. The plan is often referred to as one of "fusion"—an amalgamation into a single homogenous course of subjects hitherto treated in distinct courses; and we have heard much of "water-tight compartments." What has actually taken place in such institutions, however, has been a modification in the order in which the various topics of the mathematics of the first two years of the college and the technological school are treated. The individual topics are studied in the main as they were formerly, an individual chapter in analytic geometry, for example, not being disintegrated and the unity of its method not

[1] Cf. p. 27. [2] Cf. the report of Committee X, p. 15.
[3] Report of Committee IX, p. 17.

being destroyed; but when this chapter is finished, the next topic may be a chapter in algebra or the calculus. The early application of the calculus to curve plotting, together with a broadening of the range of curves studied, and the introduction at an early stage of the approximate solution of numerical equations, both algebraic and transcendental, by graphical methods reenforced by the calculus, are two important results of such a rearrangement of topics. On the other hand, there is danger that the loci problems in analytic geometry, thorough drill in which is essential for giving the student mastery of an important and distinctive method in this science, will be slighted because they are difficult and easier things are more attractive. In fact, care is needed throughout these courses that curve tracing be not mistaken for analytic geometry, and that the student acquire under the new system, as good students did under the old, the power to use the elementary principles of analytic geometry in later work.

The courses in the calculus.—In the first course in the calculus the simple integral as the limit of a sum is introduced at an early stage, and numerous applications of the calculus to centers of gravity, moments of inertia, fluid pressures, attractions, kinetic energy, catenaries and arches, strings on rough surfaces, and the dynamics of a particle, as well as to the traditional subject of curves and surfaces—differential geometry—are taken up. It is in the course in the calculus that the convergence of infinite series and the application of power series to computation and to the development of functions are treated. Moreover, in those institutions which have a strong department of physics or engineering a thorough treatment of partial differentiation and multiple integrals with proofs of Green's ~~and Stokes~~'s theorems is indispensable. These latter subjects are not prescribed for all students, but are offered in a second course in the calculus, which is elective, though it may be prescribed for certain classes of students. Indeed, the first course in the calculus is elective in the colleges, but is prescribed in the schools of engineering.

The Perry Movement.—Much thought and discussion have been devoted in recent years to the content of the basal courses in mathematics, both in the secondary schools and in the colleges and the technological schools.[1] While the "Perry Movement" has been influential in directing attention to this question and emphasizing the need that the instruction be given in close touch with the applications, the conclusions at which most teachers have arrived are

[1] Cf. E. H. Moore: On the Foundations of Mathematics, Bulletin Amer. Math. Soc., 2d ser., vol. 9 (1902/3) p. 402, and Science, vol. 17, p. 401; W. F. Osgood, The Calculus in Our Colleges and Technical Schools, Bulletin Amer. Math. Soc., 2d ser., vol. 13 (1906/7), p. 449; Symposium on Mathematics for Engineering Students, a succession of papers published in Science from July 17 to Sept. 4, 1908.

quite different from those of the adherents of Perry's program. For it is recognized that—

in mathematical courses mathematics should be taught rather than applied science. It is generally believed that the introduction of technical applications in the mathematical classroom is advisable only when they are so elementary and familiar as not to destroy the integrity of the mathematical courses or diminish their educational value by obscuring general principles. All agree that no illustration should be used which requires a technical knowledge not possessed by the student or readily explainable to him, as the introduction of unfamiliar concepts merely offers the student two difficulties to surmount instead of one. The chief difficulty seems to be in finding technical illustrations which are sufficiently familiar, as the mathematical courses are confined to the first two years of the curriculum, and thus precede the student's professional training.

In one instance the objection is raised that by enlarging applications the concrete tends to fill so large a place in the mind that the principle in hand is obscured as one of general application. The general opinion, however, is that if it is possible to arrange courses in parallel, so as to furnish material for illustration, a training in formal mathematics in connection with its technical applications gives depth of understanding, strength of grasp, and freedom of use most conducive to the production of efficient and progressive students.[1]

It is only fair to add that a score of years before Perry began his campaign the courses in the calculus in Harvard College were being modified with reference to bringing them into close touch with physics, and that but a few years later a course, following these, in the Newtonian potential function and the differential equations of physics, was regularly offered.

Cooperation between the department of mathematics and that of engineering.—At the time when the first widespread demands for technical instruction came to the technical schools and the colleges offering technical instruction, the departments of mathematics, like those of engineering, were unable to meet these demands. In those institutions in which both a department of engineering and one of mathematics were present, it often came about that the engineers preferred to have their students taught mathematics by men appointed in the department of engineering. While this arrangement still exists in some institutions, there is—

substantial agreement that there is no reason why engineers should be given essentially different elementary training in mathematics from others, but that all mathematical training is simply a question of teaching the subject thoroughly and in such a way that the student shall be able to use it freely. From the standpoint of mathematicians separate departments of mathematics for different classes of students are regarded as narrowing and hurtful to all concerned, and regret is expressed that specialized professional courses are occupying an increasingly large place in the curriculum to the exclusion of a more thorough grounding in broad fundamentals.[2]

[1] Report of Committee IX, p. 37. [2] Ibid, p. 38.

While this is encouraging, much remains to be done. If the best results are to be obtained, it is not enough that the mathematicians cooperate with the engineers; the engineers must cooperate with the mathematicians. The engineer who in his youth himself had inadequate training in mathematics and who has later acquired by himself a working outfit is usually not well informed regarding the best methods of the day in mathematics. His methods are the best for him because he has devised them himself.[1] But they are frequently not the best for others. Moreover, the engineer not infrequently demands, not that the student shall have the knowledge of mathematics that will enable him to understand the mathematical treatment that is to be presented to him, but that he have that knowledge in the precise form in which the engineer wishes to use it. As well expect to train a physician by giving him a course in druggist's formulas. It is not until the engineer shall avail himself of the best mathematics of the present day and be willing to make its concepts and its methods his point of departure that the problem of cooperation will have been solved.

Classroom methods.—The classroom method most commonly in use in the prescribed courses in mathematics is that of instruction in small sections of 15 to 25 students. Sometimes the hour is devoted to an oral recitation; sometimes the students solve problems assigned them in the class, either at the blackboard or as written exercises worked out during the recitation hour. The use of a textbook is general, and in some cases the instruction amounts to little more than the assignment of a lesson with some comments by the instructor, the class then proceeding to work problems as above described. For the lame and the lazy this plan has much to commend it; for a textbook can be found with exercises so well "graded" that it is merely a question of applying a rule printed in heavy type or substituting numbers in a formula. And every student can be required to do at least so much. But there is little in it to kindle enthusiasm or inspire a love for mathematics. It curtails an adequate treatment of the subject matter of the course on the part of the instructor, and it restricts the problems set to such as are so easy that they can be done at sight.

Another method consists in the instructor's setting forth the main principles of the subject in an informal talk, freely interrupted by questions from the students. Problems both easy and difficult are assigned to be worked independently outside of the class and handed in. These are corrected and returned at the next meeting of the course, and those that have presented any general difficulties are

[1] It is a common experience in a class in geometry that the solution of an "original" obtained by a pupil is intricate and cumbersome, and yet it seems clear to the pupil, for he has discovered it.

taken up before the whole class. The advantages of the method are that the student gains perspective through the classroom instruction, sees where to direct his efforts, and acquires power by working, in addition to many easy problems, some that can not be done at sight.

A further method is to some extent a combination of the two above mentioned. Two consecutive hours are devoted to each meeting of the course. The instructor uses as much of the first hour as seems desirable for the exposition of the subject, for working problems, and for general discussion. Problems are then assigned to be worked by the students in the class, and the instructor and a corps of assistants give individual aid, which frequently consists in a brief suggestion relating to a specific difficulty. The instructor is thus kept in close touch with the class and has positive assurance that the students can work the easier problems. In assigning more difficult ones, whether to be worked in the class or out of the class, he can choose with greater security such as are commensurate with the powers of the student.[1] Under the last two methods larger sections are possible, and thus the older and more experienced members of the staff are enabled to teach a greater number of students.

Whatever the method employed be, written tests are frequent. Moreover, the instructor is accessible to the student, and the student who seriously desires to learn has no difficulty in getting personal attention.

UNDERGRADUATE ELECTIVE COURSES.

Under this title are comprehended those courses which are usually taken just after the first course in the calculus or simultaneously with it, namely:[2] (*a*) Modern geometry; (*b*) mechanics; (*c*) second course in the calculus; (*d*) differential equations; (*e*) determinants and the theory of equations. To these may be added descriptive geometry and surveying. In technological schools some of these courses are prescribed for certain classes of students.

The foregoing list includes the courses which are offered by the stronger colleges and technological schools. In institutions with a highly developed graduate department the list is supplemented by further courses.

Geometry.—In a number of our colleges the courses named above are characterized not only by thoroughness of treatment but by breadth of view. The course in geometry which follows the introductory course in analytic geometry is usually not confined to an analytic or to a synthetic treatment of projective geometry, but avails itself freely of both methods, and is sometimes devoted in

[1] Cf. report of Committee IX, p. 40.
[2] Cf. report of Committee X, pp. 12 and 17, and Committee XII, p. 27.

part to the geometry of inversion, or reciprocal radii. The elements of this subject may with profit be included in more college courses, both because of the deeper insight into geometry thus afforded and for a better understanding of projective geometry itself.

The scope of the course in geometry is indicated by the fact that it usually includes abridged notation, homogeneous coordinates, and the principle of duality, as well as projection and collineation. Some treatment of lines and planes in space, and the simpler properties of central quadrics generally find a place somewhere in the program. In certain institutions with a strong department of mathematics the subjects named above are scattered through so many courses that it is impossible for a student to get a comprehensive view of the elements of geometry unless he specializes heavily in this field. A rearrangement of the offering in geometry would here seem to be desirable.

Mechanics.—In mechanics the object is primarily to teach mechanics, not analytic geometry and the calculus, though the course avails itself adequately of both these aids. Statics includes the elements of graphical statics; and in dynamics not merely the motion of particles—oscillatory motion, motion in a circle, and motion in a resisting medium—but the motion of a rigid body in two dimensions is treated. The conceptions of work and energy are developed from the mathematical side and fully illustrated in their bearings on physics. On the other hand, if the course is to be effective as an introductory course in mechanics of the kind the physicist and the engineer need, it is not feasible to go into the higher principles of mechanics. These are treated in a second course, for which the second course in the calculus is required. The value of such a broad introductory course as a part of the training of the physicist and the engineer is now pretty generally recognized in the stronger departments of physics and engineering—a marked contrast in the case of the latter departments to the situation of two or three decades ago (or even more recently), when mechanics for the engineer usually meant the strength of materials and graphical statics. But its value for the college student is still not recognized as generally as is desirable. It should find a place in more of our colleges.[1]

Algebra.—While the theory of equations, as given in the older English textbooks, usually is included in the list of courses of a college or a technological school which is able to offer some of the elective studies above named, algebra from a modern standpoint can hardly be said as yet to have gained a foothold in our colleges. "Modern higher algebra" means in this country, or has meant until recently, the theory of the algebraic invariants of the group of linear transformations, and as such is important for the geometer, but not for

[1] Cf. report of Committee XII, p. 27.

the physicist or the engineer. On the other hand, the theory of linear dependence, with its application to simultaneous linear equations, and the elements of the theory of quadratic forms are important both within and without the realm of pure mathematics. Moreover, the theory of polynomials, including resultants and discriminants and the elements of the theory of polynomials in several variables, not only has varied applications in a broad field of mathematics but a knowledge of this, as well as of the other algebraic subjects named above, is essential for the teacher of mathematics, whether his work lies in the college or in the secondary school.

When once secondary teachers are well trained in these subjects the problem of teaching school algebra will present less difficulty. Not that the present school course will be seriously modified in content, but the teacher who is equipped with a wide knowledge of the science of algebra, who has mastered the systematic and general treatment of the problems that have their beginnings in school algebra, and who has learned to know the methods of algebra by actually working with them in more advanced fields, will have a better appreciation of the importance of algebra as a science and will teach in this spirit in the secondary school instead of laboriously multiplying so-called practical problems in the belief that such problems constitute the sole justification for the teaching of algebra. Such problems are unquestionably valuable and should continue to occupy an important place in the teaching of the subject, but they should not be regarded as the one barrier that stands between algebra and its abolition from the school program.

ADVANCED INSTRUCTION.

The beginnings of advanced instruction in mathematics in this country go back to the middle of the past century, when a group of young men took up the study of mathematical astronomy in Cambridge under Benjamin Peirce. At about this time, too, the Lawrence Scientific School was founded as a department of Harvard University and, under Louis Agassiz, Jeffries Wyman, and Asa Gray, offered advanced instruction in science. It was also in these years that provision was made at Yale for graduate students to pursue their studies in special fields, including that of mathematics, and somewhat later Willard Gibbs received an appointment at that university.

Not until the eighties, however, after the beginning of the period of the enormous increase in the material prosperity of the country, are any large successes to be recorded. Three causes were at work in introducing the study of advanced mathematics into this country, namely: (1) Study abroad, (2) the foundation of Johns Hopkins

University, and (3) the elective system. How these causes operated is traced at the beginning of the report of the Committee on Graduate Work.[1]

Aims and methods of advanced instruction.—The purpose of advanced instruction has been well defined as fourfold:[2]

I. To impart knowledge.

II. To develop power and individual initiative.

III. To lead the student to express adequately and clearly what he knows.

IV. To awaken the love of knowledge and to impart scholarly ideals.

Probably the advanced instruction of this country is strongest at the present time in meeting the first of these requirements. In a number of the leading departments of mathematics courses introductory to important fields have, in a long series of years, been elaborated, which present in well-digested form a large body of material,[3] and the student is thus enabled at the outset to make rapid progress in acquiring knowledge. He is encouraged, moreover, to present a paper at the mathematical club or in the seminary, sometimes in a course that he is taking. This work is done under the guidance of the instructor, who thus has the opportunity of putting the student on his own resources and bringing him to extract knowledge from difficult sources—the original memoir or a monograph of a former age.[4] It is here, too, that the training in clear and incisive presentation, rightly insisted on in the reports that follow,[5] can be given. Clear presentation must be preceded by clear thinking and if the topic is well chosen by a careful weighing of values and the forming of a true perspective. For the student of scientific talent such work leads to the threshold or even into the realm of research, while for the student whose usefulness will lie chiefly in the field of school or college teaching it affords most valuable training.

From these facts it will be seen that the graduate student of the present day, whose preliminary training at the time he enters on work of university grade is set forth below,[6] is not compelled to go abroad for an opportunity to learn advanced mathematics. It may safely be said that, so far as the question of acquiring knowledge in the chief fields of advanced mathematics is concerned, he will be better off if he goes to a strong American university. The arguments

[1] Committee XII, pp. 6–10 and 21.

[2] Report of Committee XII, p. 12.

[3] The instruction in these advanced courses, and to some extent in elementary courses, is given by lectures, which, however, have little to do with a system supposed to exist and indefinitely described as "the lecture system." How the instruction is actually conducted when lectures are used is set forth in the report of Committee XII, pp. 13, 31, and 32.

[4] Report of Committee XII, p. 51.

[5] Ibid., pp. 17, 47, 51, and 52.

[6] Ibid., p. 10, sec. 2.

in favor of foreign study are well set forth in the report of the Committee on Graduate Work.[1]

As a further aid to the attainment of the end described under II may be mentioned the use of problems in the advanced courses.[2] While it is less easy in these courses than in the elementary ones to find numerous problems by which to test how far the student is assimilating what he is receiving, an instructor who for a long series of lectures finds himself giving out no problems to be handed in the next day or in the near future (not at the end of the term) should scrutinize the reaction of the class to his instruction.

It is pointed out in the report of the Committee on Graduate Work that in a number of graduate schools graduate students are exempt from examinations till just before they come up for a degree. If examinations are framed in such a way as merely to test what the student knows (or does not know), such a procedure can be explained; for graduate students of mathematics are, almost without exception, characterized by high ambition and a fixed purpose, and the faithfulness of their work need not be tested by frequent examinations. But an examination, when properly conducted, is much more than a mere test; it contributes constructively to the instruction in the course and thus becomes an important aid to the graduate as well as to the undergraduate.

Aside from briefer tests which may be held at any time as the course advances, there are three kinds of examinations in use in this country—the written examination, held under supervision at the close of each half year and usually occupying three hours; the thesis; and the "long paper." The latter consists of a dozen to forty (or even more) questions of substantial character, usually distinctly more difficult than the daily exercises, assigned toward the close of the half year to be worked outside of the class in a period of a few weeks. The students are put on their honor not to get help from one another, but they may consult books and their lecture notes without restriction. For the purpose of developing power this system has few equals.[3]

In connection with this subject, the "quiz" should also be mentioned.[3]

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[1] Committee XII, general report, pp. 18-20, sec. 5.

[2] Collections of such problems have been published from time to time in the *Annals of Mathematics*; cf. Bôcher, Examples in the Theory of Functions, *Annals of Mathematics*, 2d ser., vol. 1 (1899-1900), p. 37; Bouton, Problems in the Theory of Continuous Groups, ibid., p. 93; Osgood, Problems in Infinite Series and Definite Integrals, with a Statement of Certain Conditions which are Fundamental in the Theory of Definite Integrals, ibid., 2d ser., vol. 3 (1901-2), p. 129.

The English textbooks, as Forsyth, Theory of Functions, and Bromwich, Infinite Series, also contain long lists of problems, which, however, could be rendered more effective by pruning.

[3] Report of Committee XII, pp. 32 and 50.

On the other hand, it must not be inferred from what has been said above that the advanced student of scientific ability is kept in leading strings throughout his whole university course. He usually plans his work for the coming year in consultation with a professor with whom he stands in close relations, and when he once has entered on a piece of real investigation he has the utmost freedom in the use to which he will put his lecture courses. It often happens that he attends some courses merely as a hearer; in others he follows the treatment carefully, but does not make the final review that would be necessary if he were held to the examination at the end of the course. For it must be remembered that the doctor's degree is not based on a count of courses, but on attainments in the main fields of mathematics and on a thesis.

The highest of all the purposes named above, and the most difficult to attain, is the last: To awaken the love of knowledge and to impart scholarly ideals. Our people are not deficient in their innate love of knowledge, even in this age of the exploitation of playthings. Aside from these things, however, we find widespread within the college, as in the community, the doctrine of intellectual socialism—a desire to learn only those things by which one can minister immediately to the needs of the many. Devotion to science is looked on as selfish in an age marked at once by selfishness and a quickened conscience. Little wonder that with the law and business on the one side, and the ideal of immediate social service on the other, the interests of productive scholarship fare ill.

The general report of the Committee on Graduate Work[1] has done well to signalize two needs of the present day—*intelligent* idealism and high *scientific* standards. To possess these things as a guide of life, to have the intellectual gifts that are necessary for achievement, and to interpret these things to the student by sympathetic interest in his work and faith in the possibility of his attaining to the larger achievement which persistent and intelligently directed efforts assure to one of good ability, is to be a great teacher.

It is fitting that in a report on the recent history of mathematics in the United States the debt which this country owes to Germany be gratefully acknowledged. At a time when there was not an important mathematical center on this continent, Americans were attending lectures and working in the seminaries of the leaders of the science in Germany. The ideal of scholarly achievement was manifested in the lives of these men, and the university atmosphere was filled with the traditions of the great men whose life work—sometimes hardly closed—had changed the face of mathematics. To these Americans, as to their German Kommilitonen, the doors of the Seminar were

[1] Committee XII, pp. 13 and 18.

thrown wide open, and not a small part of the influence under which these men came is to be found in the zeal with which intellectually strong men among the German students laid hold of the opportunities which the whole German system offered. Here, indeed, the conditions were favorable for stimulating the love of knowledge and imparting scholarly ideals.

THE UNIVERSITIES OF THE UNITED STATES.

The term "university" is used in this country in two distinct senses. It is applied to the higher institutions of learning, which comprise the four faculties, and the stronger ones of which rank with the universities of Europe. It has also been appropriated by smaller institutions, some of which do not even rank with the better colleges. Between these two extremes lie many of the State universities; for while some of these, like Wisconsin and California, belong distinctly to the former class,[1] there are others whose department of mathematics does not number on its staff a single mathematician of scientific standing, nor is it engaged in giving advanced graduate instruction.

Among the strongest institutions are to be found certain of the endowed universities, some of which grew out of the colonial colleges,[2] while others, of more recent origin, began with graduate departments.[3]

The older colleges of the country were founded by private individuals or religious bodies, and were originally to a greater or less extent denominational. They are under the control of self-perpetuating boards of trustees. The State universities have been created by the State legislatures, sometimes with the help of land grants from the National Government, and are under the control of trustees or regents chosen by State authority. There is no central organization or control, custom alone making the policies of different institutions fairly uniform.

Over 30 colleges and universities in the United States offer graduate work in mathematics, but less than 15 have given a doctor's degree in that subject within the last five years. The following 24 offer courses of an advanced character and report three or more graduate students for the year 1908–9: Bryn Mawr, California, Chicago, Cincinnati, Clark, Colorado, Columbia, Cornell, Harvard, Illinois, Indiana, Iowa, Johns Hopkins, Michigan, Missouri, Nebraska, Northwestern, Pennsylvania, Princeton, Leland Stanford, Syracuse, Virginia, Wisconsin, Yale.[4]

[1] The State universities do not comprise all four faculties. Their strength lies chiefly in their departments of science.

[2] Harvard College was founded in 1636, Yale in 1701, Princeton in 1746, and Columbia in 1754.

[3] Johns Hopkins, opened for instruction in 1876, and Clark University, opened for instruction in 1889, had originally only graduate departments. Chicago, opened for instruction in 1892, had from the start both a graduate and an undergraduate department.

[4] Report of Committee XII, p. 20.

In recent years a number of Western States have made appropriations increasing in a gratifying manner the budget of their State universities. One result of this change has been to strengthen scientifically the departments of instruction and in particular to make possible the offering of advanced work in mathematics. And what has happened in one State may at any time happen in another in which civic ideals have reached the necessary stage of development.

It would be a mistake, however, to think that the lines of demarcation fall between the endowed and the State universities. If it be true that the strongest universities of the country are in the former class, some of those of the latter now rank among the foremost universities of the country, and with their increasing resources bid fair, in case they can emancipate themselves from the evils of political influence, to rival even the strongest endowed institutions.

The teaching force.—With the introduction of the elective system came the opportunity for professors, even in the smaller colleges, to offer some advanced work in mathematics. The result has been to create a demand for men better trained in their science, and thus the quality of undergraduate teaching has been improved. In the larger universities the question soon arose as to the desirability of dividing the force so that some men would have only undergraduate work and others would give exclusively graduate courses. While such a division has, in a few cases, been made, the great majority of men in charge of the advanced instruction also give courses for undergraduates.[1] The economic side of the situation need not be discussed here, since it is fairly obvious. It is, however, important to note that the system is one under which effective instruction both in advanced college courses and in courses of graduate grade has been developed, for the instructor in an advanced course becomes and remains sensible of the needs and the possibilities of his students. It is true that some men with a keen interest in advanced work find undergraduate instruction irksome; others, however, find that it affords a desirable variety in their work, and they value the wider acquaintance among the students which is thus rendered possible. The chief difficulty with the present system is in the unduly large number of hours per week which the professor is called on to give to classroom instruction, and in the administrative work with which he is burdened. We shall return to these subjects presently.

While some of the weaker institutions offer elective courses in mathematics far beyond the strength of the staff,[2] the inflation tends to subside with a real strengthening of the department.[3] Moreover,

[1] Cf. report of Committee XII, pp. 22 and 42.
[2] Report of Committee XII, p. 27.
[3] Ibid., pp. 13 and 14.

through the system of alternation in the courses given, an instructor offering different advanced courses in successive years, a broad field is covered even when the staff is small.

About half the instructors in charge of the advanced instruction have studied abroad, chiefly in Germany.[1] Of the main fields of mathematics, analysis is more strongly represented than geometry, algebra, or applied mathematics. Indeed, a strengthening of the latter field is much to be desired. But a premeditated separation of pure from applied mathematics has not taken place, and there is apparent a well-balanced offering of courses in the principal fields, so far as this is possible.[2]

With the great increase of recent decades in the number of students attending the colleges and other institutions of like grade, there have arisen new administrative problems. In the absence of any central organization and with conditions varying widely in different parts of the country, each institution has been obliged to work out its own solution. The work of the dean and of faculty committees dealing with such questions often falls to men who have proven themselves to be efficient teachers. Moreover, with the large amount of personal supervision of the students common in American colleges, the labor of administration in departments of mathematics numbering several hundred students is very great. Such administrative duties have seriously interfered with the scientific work of American professors, and have not infrequently made it impossible for them to find scholarly leisure for the enjoyment of their subject, without which the best teaching can not thrive.

If this is bad at the top of the ladder, it is even worse at the bottom. Many a young man fresh from his university studies begins his teaching career with an exorbitant number of classroom hours per week, only to find that the prize of advancement in the institution at which he is located is more easily won by the development of administrative capacity than by the production of scientific papers of real value. It is true that these strictures do not apply to the strongest departments of mathematics in the country. As is pointed out elsewhere, the demand for first-rate men is too keen and the supply too restricted for it to seem best to these departments to put young men of promise to such uses. But between the really strong and the very weak institutions there is a wide belt of colleges, technological schools, and universities, in which such practices are common. On the other hand, it is not enough to develop only the exceptional men. There must be a large constituency of men of good ability whose environment is such that they can develop the powers which nature and a good university training have given them. The interests of

[1] Report of Committee XII, p. 23. [2] Ibid., p. 29.

true scholarship demand that the energies of these men should be conserved for teaching and so far as may be for research. The two things go hand in hand. The teacher who once comes to feel that he has his courses in final form, from that time on declines in the quality of his instruction. It is only through the inner satisfaction that comes from learning more of his science and if possible contributing something to its progress that he can keep his instruction fresh and virile, and give to his work in the classroom the zest that kindles a love for knowledge in the breasts of his students.[1]

THE MASTER'S AND THE DOCTOR'S DEGREE.

The requirements for the master's degree almost universally consist in at least one year's work beyond a bachelor's degree granted by an institution of good standing. The work must be largely in one field, as, for example, in mathematics. But even as much as one-half may lie in a field more or less closely related, as, for example, physics, engineering, astronomy, or education, when mathematics forms the chief field. The work is tested by examinations, and a higher standard must be attained than is ordinarily required of candidates for the bachelor's degree. A thesis is often required, but it is not expected that the thesis shall embody research by the candidate.[2] Graduates of the weaker institutions of collegiate grade are freely admitted to enrollment in the graduate schools of the leading universities. But they are required to make up specific deficiencies, or otherwise to do additional work, before an advanced degree is conferred.

In 1904 a committee reported to the Chicago section of the American Mathematical Society on the requirements for the master's degree, with especial reference to conditions in western institutions. Their findings are summarized in the report of the Committee on Graduate Work.[3]

For the doctor's degree a distinctly higher standard is required and enforced. In all American universities of good standing it is distinctly a research degree. In several of the stronger universities it has a standard at least as high as the best German standard. "The requirements for the doctor's degree in universities which have given it to any extent during the last 10 years are tolerably uniform,[4] but in this matter so much depends on the unwritten standards of individual professors or departments that there still remains a great

[1] Cf. report of Committee XII, p. 55.

[2] Ibid., pp. 14, 32, 54. The master's degree is also conferred as an honorary degree under wholly different conditions. It is only the degree as conferred "in course" which is here considered.

[3] Committee XII, p. 34.

[4] Cf. report of Committee XII, subcommittee 2, p. 36.

difference in the ease with which the degree can be obtained at different institutions. It is for this reason that the suggestion which is sometimes made that it would be well to attempt to formulate definite standards for the doctor's degree, to which the universities of the country should conform, seems to be of slight practical value."[1]

It is fair to say that the requirements for the doctor's degree in mathematics in the stronger universities are: A bachelor's degree, the ability to read French and German,[2] the equivalent of three years' study in the field of advanced mathematics, certain requirements of "minors," and a thesis of substantial quality, embodying the results of the candidate's research. The publication of the thesis is obligatory in most universities.

The time for effective study.—We use the word "effective" with reference to both material and intellectual results. The appointing powers in our institutions of higher learning are well aware that a large part of the instruction is given by young men on small salaries. Except in the case of extraordinary success as an investigator or as a teacher, men who have reached the thirties are not called to other institutions. The reason is in part economic, lying in the fact above referred to. If an institution is later to pay the increased salary of an advanced appointment, it wishes in a measure to recoup itself by also getting the services of the teacher at a time when these are less expensive. Hence it is in the material interest of the teacher to complete his university studies without interruption immediately after his college course, in order that he may find early the institution in which he is to be advanced.

Fortunately the material and the intellectual interests of the teacher are not in conflict at this point, for it is a fact that can not be too widely proclaimed that it is only in youth[3] that the successful study of mathematics with reference to scientific productiveness can be begun. The teacher of 30 who has not as yet carried his study of mathematics beyond the college courses can, it is true, acquire useful knowledge and greatly improve his efficiency as a teacher by a year or two of graduate study spent at a mathematical center. But the days when it was still possible for him to prepare himself to contribute to the science have passed. Hence every effort should be made by those who advise young men of distinct scientific promise to guard them against the danger of letting the early years slip by in teaching.

How little the youth of to-day realize these facts is clearly brought out by statistics given in the report of Committee XII.[4] By far

[1] Cf. report of Committee XII, p. 14.
[2] Italian may also be necessary.
[3] Ibid., pp. 11 and 55.
[4] Ibid., pp. 25, 26.

the greater number of the students who are doing advanced work in mathematics are supporting themselves wholly or in part by teaching. At a time when they should be devoting all their energies to securing for themselves the highest place among the future men of science which their native ability enables them to attain they sell their birthright for a mess of pottage. Inspired by the best of motives—the desire, now that they have arrived at the years of manhood, no longer to be a pecuniary burden to their parents, who often have made great sacrifices to put them through college—they are ignorant of the fact that the opportunity which nature offers them will soon be withdrawn if not at once laid hold on and exploited to the utmost. They see about them men who have risen to the highest college or university positions, but who have now, perhaps long since, ceased to do scientific work. If these men have still retained something of their youthful ideals pertaining to productive scholarship, they tell themselves and others that administrative or classroom duties prevent them from continuing in scientific work. That they could resume such work again at will is taken for granted, and thus youth is still further misinformed regarding the facts of nature.

The career of learning and advanced instruction.—In foreign countries the career of college or university professor attracts the best youth of the nation. In England, in France, in Germany, no calling is more honorable and dignified nor does any more strongly incite the intellectually endowed to put forth their best efforts to secure its prizes, and to undergo if need be years of material privation in qualifying for the supreme test—that of scientific achievement. Among our people the prizes of this calling are not clearly discerned. We have no word that describes the career; for *college* or *university teacher* suggests to the layman the nurture of immature minds, not the pursuit of knowledge and the rewards of discovery, close personal relations with gifted youths, and the fellowship of educated men. Moreover, the freedom of life which the college or university professor enjoys—the opportunity to do those things which are pleasantest to a man of strong intellectual life and warm sympathies, with a minimum of drudgery—is not generally recognized in the community at large.

And so we must record the fact that in the United States the law, engineering, and business are powerful competitors when the young man comes to choose his profession. "The tendency so strong in our day and country to regard the man of action as being of nobler clay than the man of thought and ideas, reenforced by the much greater financial prizes open to the former, whether he be lawyer, business man, or engineer, creates a situation where it is ~~not~~ easy to secure for mathematical study a due proportion of the strongest youth in our college communities." [1]

[1] Report of Committee XII, p. 12.

APPOINTMENTS AND DEGREES.

We have said much that pertains to the training of teachers. We wish now to address a word to those who have to do with the appointment of teachers.

At the time that the modern movement in mathematics set in, three decades ago, many of the men who became leaders had at the close of their student days attained the degree of doctor of philosophy. It was natural that, as the movement spread, those institutions which wished to strengthen their departments of mathematics should look to men of sound scientific training and interest in research. And so it easily came about that the doctor's degree was accepted as standing for these things, and in some institutions it has been made a requirement for appointment or advancement. It is, indeed, highly desirable that the departments of mathematics be manned by mathematicians who are thoroughly trained, are good teachers, and are actively interested in research. The fact confronts us, however, that while there is a small steady output of first-rate mathematicians, the supply of such men is far behind the demand for them. If many times the number of men who now enter on the profession of academic instructor were to choose this field, it would still be possible for the capable youth to achieve success.

The result is that many departments of mathematics, while exerting themselves to secure men of first-rate scientific ability, find it impossible to make the appointments they wish. To justify the appointments they do make, the holding of the doctor's degree by the candidate goes a long way. Now, it is desirable that every student enrolled at a university that has a strong department of mathematics be given opportunity and encouragement to put forth his best energies in the attempt to do original work, and that he be not discouraged if he does not at once meet with success. "The present sharp differentiation of college mathematicians into two classes, the holders of the master's and of the doctor's degrees, is in many ways a most unfortunate one. Many men have stopped with the former who are capable of proceeding much further. The committee wishes to record its emphatic belief that every man should be encouraged to study just as far as his ability and taste may qualify him, without stopping at the line of either degree. Many a teacher without the kind of ability necessary for research can yet be encouraged to become a thorough scholar in some definite line, instead of looking to administrative office as his career."[1]

This is the aim which the departments of mathematics in the field of graduate work should have before them—to give to each student the best training he can receive for the work that he has the natural

[1] Report of Committee XII, p. 55.

capacity to do. It is also their duty to help him discover what that work is; and in case, on fair trial, it becomes clear that he is not qualified for the work of research, to cease to help him to obtain the doctor's degree. The great stress, however, which the appointing powers in some colleges and technological schools lay on the holding of the doctor's degree is subversive of the best interests of the future teacher. It has already led to the fostering of spurious research and set false ideals before men who might otherwise have become effective teachers.[1] Nor can it be defended on the ground of difficulty in getting information regarding students available for appointment. A personal letter of inquiry stating what sort of appointment it is desired to make and addressed to the head or to a member of the department of mathematics at a leading university—a number of universities maintain a Bureau of Appointments for the purpose of dealing systematically with such inquiries—brings out a frank answer, for it is recognized that no one's interests are served when a candidate is appointed to a position which he is not qualified to fill.

In the case of the master's degree there exists a similar pressure from the schools.[2] This degree, as conferred by the stronger institutions, has a fairly definite meaning. It does not stand for research, but it means proficiency in mathematics distinctly beyond the first course in the calculus, and it often is given on the basis of really advanced work performed with high credit. But it may well happen that a graduate student—for example, a school-teacher on leave of absence—can profit more from a year of study made up in part by broad undergraduate elective courses, such as those described on page 45, and including a course in physics or education. To make the program conform to the requirements for the master's degree, the student must sacrifice a course adapted to his needs for one which it is beyond his powers to do profitably, or the standard of the degree must be lowered. Both alternatives are unfortunate alike for the student, the university, and the future employers of the teacher. The professor who advises the student regarding his choice of courses has in view the program of study from which the student can profit most, and it is unfortunate when this program is seriously disturbed for the sake merely of obtaining a degree.

It is felt by some that the instruction in our graduate departments is fashioned too much with reference to the needs of those students who are to devote themselves to research and too little with reference to the needs of prospective teachers.[3] Both classes of students meet on the common ground of the courses of the middle group—undergraduate electives and first-year graduate courses. It is important

[1] Report of Committee XII, pp. 16, 17. [2] Ibid., p. 82. [3] Ibid, p. 54.

that these courses be both broad and deep. There is no tendency to go to the extreme of making them superficial, merely surveys of the field; but specialization both in content and mode of treatment should be guarded against in the interests of both classes of students, especially those who will not pursue the study of mathematics at the university much further.

For the teachers in secondary schools and the smaller colleges much can be done through the summer schools. As yet, with few exceptions, the mathematics offered in the summer schools is of elementary character, often consisting merely of the subject matter of the school course. On the other hand, when professors of mathematics plan a course for school-teachers they are often tempted to turn to the formal side—the axioms and philosophical questions. What is needed, however, is courses of substantial mathematical content, like the undergraduate electives described above, for without a thorough knowledge of the subject matter of mathematics teachers can not gain real power to improve their teaching.

While these are primarily questions for the department of mathematics offering the courses, so much depends for the efficiency of the future teacher on the policy of that department that the appointing powers will do well to cooperate with it in its efforts to secure for each individual student the course of study best adapted to his needs. They can do this by basing their judgment of the training of the candidate on his course of study and his success in the course as attested by the men with whom he has worked, and not looking merely at the degree obtained. While they can not, as a rule, go into the subjects in detail which have made up the course, they can ascertain the quality of the candidate as regards his ambition and his ideals, his industry and perseverance, the degree of his advancement and his success in scientific work, and the direction in which his greatest strength lies.

The above procedure is followed at the present time by the larger colleges, technological schools, and universities. It may well be adopted by any institution which is able to offer a position that a well-trained candidate will consider.

CONCLUSION.

Among the more important conclusions to be drawn from the foregoing report on mathematics in the higher institutions of learning, and the committee reports on which it is based, are the following:

The teaching of mathematics in the colleges and other institutions of like grade shows progress, and the departments of mathematics in these institutions have their problems well in hand.

There is increasing confidence and a spirit of cooperation between the departments of mathematics and those of applied science as regards undergraduate instruction.

For the continuance of progress in improving undergraduate teaching a much larger supply of well-trained men for the college staff in mathematics is needed.

It is in the interest of good teaching that professors be not overburdened either with administrative work or with classroom hours. Moreover, improvement in these directions will make the career of college professor more attractive and thus more men of the right type will turn to this career.

Advanced instruction, as given in formal courses and seminaries, is broad and thorough.

A few of the strongest departments of mathematics offer opportunities for research comparable with those of European universities. Both the offering and the demand for such opportunities fall, however, in amount far short of what one should expect to find in America.

There is, nevertheless, at the present time an opportunity for young men of first-rate scientific ability and aggressive scholarship to find in the university career a life work full of things that make for happiness—the enjoyment of productive intellectual activity and stimulating association with colleagues.

That young men of this type are beginning to respond to the more favorable conditions of recent years in the strongest universities is shown by the appointments made at these universities and by the output of high-grade scientific papers published in the leading mathematical journals, notably in the *Transactions of the American Mathematical Society*, during the past decade.

APPENDIX.

HISTORICAL SUMMARY OF THE WORK OF THE INTERNATIONAL COMMISSION.

ORGANIZATION.

The section on philosophy, history, and instruction of the Fourth International Congress of Mathematicians, held at Rome, April 6 to 11, 1908, after listening to a series of reports on the teaching of mathematics in the principal countries, decided to submit to the congress a resolution to create an international commission to make a general study of the progress of mathematical instruction in the various nations. The suggestion was indorsed by the congress, which at the meeting of April 11 adopted the following resolution:

"The congress, recognizing the importance of a comparative examination of the methods and plans of study of the instruction in mathematics in the secondary schools of the different nations, empowers Messrs. Klein, Greenhill, and Fehr to form an international commission to study these questions and present a general report to the next congress."[1]

A central committee was later organized as follows: President, Professor F. Klein, Göttingen; vice president, Professor Sir George Greenhill, London; general secretary, Professor H. Fehr, Geneva.

The committee began work immediately, and at a meeting held at Cologne in September, 1908, adopted a preliminary report on the organization of the commission and the general scope of its work.

THE DELEGATIONS.

The commission was formed by delegates[2] representing the countries which had taken part in at least two of the international congresses of mathematicians with an average of at least 2 members. It was arranged that each of these countries should have at least 1 delegate, and that those countries which had had an average of at least 10 members should have 2 or 3 delegates.

The different delegations were invited to affiliate with themselves national subcommissions comprising representatives of the various stages in the teaching of mathematics in the general schools, and also in the technical and professional schools. These subcommissions were designed to aid the delegates

[1] The next congress will be held at Cambridge, England, in August, 1912.

[2] In the English language a member of a commission is called a commissioner, and the American members of this commission have so styled themselves. In French similar usage does not obtain, the French word *commissionaire* having quite a different meaning. Therefore in *L'Enseignement Mathématique*, the official organ of the commission, the members, although not delegated by their several countries, but appointed by the central committee, are called—*faute de mieux*—*délégués*.

in the elaboration of the reports, and in the United States they became the committees and subcommittees that furnished reports upon specific subjects.

FINANCIAL MATTERS.

As the Fourth International Congress did not furnish any funds, the Governments of the participating countries were invited to place at the disposition of their delegations a sum sufficient to cover the expenses of the delegations and of the national subcommissions and to contribute to the general expenses of the commission.

To provide for the general expenses of the commission (including notably expenses of the general secretary and the central committee), a fund was formed by contributions from each of the participating countries. In the United States there was no fund upon which the Bureau of Education could draw for such an investigation, and so the work has necessarily been carried on under great difficulties. Had it not been for the generous support of a few public-spirited men and institutions, it would have been impossible to accomplish anything worthy of the country, but by means of this support a series of bulletins has been prepared setting forth the work that is done in our schools. Of the efforts of all those who have come to the support of the movement, the commissioners wish to express in this public manner their sincere appreciation.

PUBLICATION OF REPORTS.

The question of the publication of the various American reports was happily settled through the cordial cooperation of the United States Bureau of Education. Through the encouragement and support of ex-Commissioner Elmer Ellsworth Brown, LL. D., now the chancellor of New York University, and of his successor, Commissioner Philander P. Claxton, Litt. D., it has been possible to publish the reports through the bureau and to distribute them gratis to all who cared to ask for them.

The commission has arranged with Georg & Co., of Geneva,[1] to sell most of the foreign reports. Copies of the reports of the American commission may be obtained from the Bureau of Education, Washington.

Besides this publication of the reports, a summary of the progress of the work has been given from time to time in the official American organ, *School Science and Mathematics*, in the official organ of the commission, *L'Enseignement Mathématique*, and in other journals.

GENERAL AIM OF THE COMMISSION.

The general aim of the commission was formulated by the central committee as follows:

To make an investigation and publish a general report on the present tendencies of the teaching of mathematics in the various countries.

It was decided that regard should be paid not only to the methods of instruction and course of studies, but also to the general scheme (organization) of studies, without, however, giving a complete historical development of the same. It was agreed that it was not the purpose of the commission to elaborate statistics. It was urged that the work of the commission ought to make evident what are the general principles which should inspire the teacher, rather than to seek uniformity of details or to propose programs which should be adapted at the same time to the institutions of the various countries.

[1] Address Georg et Cie., Editeurs, 10 Corraterie, Genève, Switzerland.

ORGANIZATION OF WORK.

In order that the investigation should achieve results really useful to the progress of instruction, the central committee expressed the desire that all the delegates and their national subcommissions collaborate actively and devotedly. The committee suggested that the principal points of the various reports be discussed at educational gatherings and in scientific societies that are interested in the progress of the teaching of mathematics.

Although the text of the resolution of the congress at Rome mentioned only the teaching of mathematics in secondary schools, it was seen that the aim of these schools and the length of their courses vary so much in different countries that it was necessary to extend the work to include the whole field of mathematical teaching from the lowest stages to the highest. It was therefore decided that the commission should not confine itself to the institutions leading to the university, but should study also the teaching of mathematics in technical and professional schools. Because of the growing importance of these schools and of the new requirements which are continually made on mathematical instruction, it was felt to be necessary to accord a large place to applied mathematics.

GENERAL PLAN OF THE WORK.

The central committee laid out a general plan for the work in the various countries, and this has been followed in spirit, if not in detail, by all who have taken part in the work.

The plan contemplated not only the consideration of the present state of the organization and the methods of mathematical instruction, but also the modern tendencies in the teaching of mathematics.

Under the present state of the work it was proposed first to consider the various types of schools, giving a concise exposition of the various public institutions of learning in which mathematical instruction is given and the aim of each school, including schools for girls as well as for boys.

The institutions were distributed according to the following classification:

(a) Primary schools, lower and higher.

(b) Middle schools or higher secondary (lycées, gymnasien, realschulen, etc.).

(c) Middle professional schools (technica, etc.).

(d) Normal schools of the various grades (seminaries for teachers, colleges for teachers, etc.).

(e) Higher institutions (universities and technical schools).

It was thought desirable that this exposition should be accompanied by a schematic table giving a general view and making evident the succession and correspondence between the diverse establishments and indicating also the average age of the students.

The committee proposed next to consider the aim of the mathematical instruction in general and in the various branches of the science. It was suggested that the question be studied for the various types of institutions, taking into account, whenever necessary, applied mathematics, notably mechanics. The feeling was expressed that the aim of mathematical instruction varies necessarily in different institutions, and that it has undergone some transformation in the course of the last decade. It may be purely formal, or formal but taking account of intuition; it may also lay stress on logical development and the utilitarian side simultaneously; or it may regard only the practical. On the other hand, the development of the memory may be the principal aim, or contrariwise the development of the mathematical faculties.

It was also suggested that there be clear statements of the branches of mathematics taught in the different types of schools; the time alloted to these branches, the extent of the program, and the attention paid to correlation between these branches and applied mathematics (including mechanics) and physics.

In recognition of the fact that the system of examinations has a great influence on the method of instruction, the suggestion was made that the characteristics of the examinations in each category of schools should be concisely indicated, particularly of those leading to "certificates of maturity," to "degrees," etc., and that attention be given to the examinations of candidates for teaching.

Under the topic of methods, the several countries were asked to report upon the methods used in the various institutions, from the primary schools to the higher institutions; the material of instruction, including mathematical models; the use of manuals, textbooks, and collections of problems; the nature of the theoretical problems; the problems taken from the applied sciences; and the practical work demanded of pupils.

A report was also asked for upon the question of the preparation of candidates for teaching, including in the discussion the diverse types of schools, and the requirements demanded by the school authorities with regard to both the theoretical and the professional preparation.

The second part of the investigation, that having to do with the modern tendencies in the teaching of mathematics, the central committee suggested be conducted on substantially the following outline:

Section I. Modern Ideas Concerning School Organization.—Reforms in studies. New types of schools. The question of coeducation of the two sexes.

Section II. Modern Tendencies Concerning the Aim of Instruction and of the Branches of Studies.

Aim of Instruction.—New branches or new chapters to substitute for useless topics of study in the course, or those of secondary interest, but retained by pure tradition or by routine.

Considering the rapid progress of mathematics and its applications the commission proposes to take up anew the question. What are the branches of this science that are able to contribute most to general culture? Among the subjects which are now seeking a place in the elementary programs may be mentioned, on the one hand, the differential and integral calculus, analytic geometry, certain notions of descriptive and projective geometry, and a study of physics from the mathematical point of view.

It will be useful for the investigation to examine in what measure account may be taken of these demands and that it establish what is the necessary minimum of elementary geometry, descriptive and projective geometry, algebra, trigonometry and analytic geometry, the differential and integral calculus, to form a foundation for subsequent studies.

The same question arises for professional schools. What are the branches useful for the different careers?

Section III. Examinations.—Projects for the transformation of the system of examinations or for their complete suppression.

Section IV. The Methods of Teaching.—Modern ideas concerning methods at different stages of instruction and in different types of schools. Correlation among mathematical branches. Relation between mathematics and other branches. Problems and practical applications; models and instruments. The use of manuals.

On Certain Points of this Section. 1. Since the period of Pestalozzi, psychological considerations have played an important rôle in primary education and for a generation they have been useful, in a certain measure, in the

formation of programs for secondary schools. It would be well to examine what are the results of psychology applicable to the teaching of mathematics, and to what degree they are useful in the reform of this teaching. It is advisable to examine particularly the rôle of the initiatory instruction and the necessity that theoretical instruction be preceded by intuitive instruction.

At what moment, on the contrary, should purely logical considerations take the preponderance, for example, in the study of elementary geometry or in differential and integral calculus?

2. The Practical Applications.—Many schools have devoted long discussions to the part which should be assigned to considerations of practical and experimental nature.

(*a*) In elementary instruction may be mentioned, for example, paper folding, outdoor work, use of simple instruments of measuring, observational geometry, etc.; practical calculating and approximations (degree of approximation, logarithms to different numbers of places, use of the slide rule, etc.); the general question of graphs in algebra, the more extensive use of cross-section paper.

(*b*) The question of mathematical laboratories has been agitated during the last few years. What has been done in this matter and what are the results. Mathematical models made by the pupils. The place of collections of models.

What are the means which will permit mathematics to be accorded a better place in popular instruction (university extension). Place of applied mathematics in museums. Mathematical recreations.

In general, the means of reacting against the popular prejudices against mathematics should be considered.

3. Correlation Among the Different Mathematical Branches.—It will be useful to examine in what measure the conventional limits which exist between certain subjects of pure mathematics, such as algebra and geometry, elementary and analytic geometry, and geometry and trigonometry, may be made to disappear. Not only the possibility of such reform should be investigated, but regard should also be paid to the inconveniences and the dangers which may result.

It will be well, further, to know the result of the following transformations which have been proposed or reexamined during the last few years;

(*a*) The place of demonstrative geometry relative to algebra. (*b*) The fusion of plane and solid geometry. (*c*) The more intimate union of differential and integral calculus.

4. Relations between Mathematics and Other Subjects.—In the same connection it will be useful to examine the points of contact which exist between mathematics and other subjects. Thus the relations—(1) With drawing (geometric, technical, and artistic); (2) with the applied sciences; (3) with the other scientific branches (physics, chemistry, biology, geography, etc.); (4) with philosophy; (5) with the problems of daily life.

These points of contact are important for practical education. It will not be sufficient to study simply the possibilities and the general desiderata; it is necessary to take account of what is now being successfully done and of the dangers involved. For example, those who desire a close relation between mathematics and physics should show exactly what geometric notions have a direct bearing on physics and cite those problems of elementary physics which require simultaneous linear equations, equations of the second degree in one or more unknowns, irrational equations, and progressions.

5. Historical Considerations.—Demand is being made that a larger place be accorded to the historical development of mathematics. In what degree is this possible and desirable?

Section V. The Preparation of Teachers.—What are the conditions which a rational preparation of candidates for teaching should fulfill? How are the theoretic courses and the practical preparation to be organized?

The progress of teaching depends directly on the preparation of the teachers. This is a question of fundamental importance. The studies and the exigencies vary necessarily from one country to another; they depend much on the number of candidates and the facilities at hand in regard to education. Consequently the committee believes that it will be useful to take account of the reforms or the projects for reform which are now being considered with a view to bringing the training of teachers into conformity with modern conditions, not only for the personnel of primary and secondary schools but also for the university.

This inquiry should touch notably:

(*a*) The mathematical work required of candidates.

(*b*) Their introduction to scientific research.

(*c*) The best method of presenting theoretical and practical pedagogy (considered as the science of education).

(*d*) The question of the sex of the teacher in different school years.

(*e*) Questions concerning, for example, the time to be devoted to the history of mathematics, the history of the teaching of mathematics, the recreational side of mathematics, and general literature touching mathematical education.

The central committee further suggested that in each of these sections there should be emphasized concisely, on the one hand, that which characterizes the proposed reforms and, on the other hand, what are the dangers to be avoided and the objections which are made by those who oppose the proposed changes. It recommended the following as some of the fundamental questions which ought to be discussed:

1. The desire to render instruction attractive may detract from its serious character—a result which would be disastrous as much from the standpoint of the science as from that of the practical value of mathematics.

2. A misconceived psychology might lead to an exaggerated use of the logical bases of mathematics, with an attendant result of continued uncertainty on the part of the pupil.

3. The results of neglecting the abstract side, which seems necessary to fix indelibly in the mind the fundamental mathematical truths.

4. The danger of not realizing that geometry, as it is ordinarily conceived, leads to results of a nature quite different from those which are furnished by algebra, and that a fusion of the two might be followed by the loss of some of the principal advantages of each of these branches. The same for other subjects.

Still other dangers present themselves, and the commission holds that it is necessary to examine all with care, so that only those reforms which lead to real progress may be undertaken.

It was in the spirit of these general suggestions that the investigation was undertaken and carried on in the United States.

THE ORGANIZATION OF THE INVESTIGATION IN THE UNITED STATES.

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[1] Bulletin, 1911, No. 13.

Committee No. II. Special Kinds of Elementary Schools.[1]

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[1] Same bulletin as Committee No. 1.
[2] Bulletin, 1911, No. 16.

Subcommittee 3. Coeducational High Schools in the East.
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William H. Norton, Boys' Latin School, Boston, Mass.

Subcommittee 9. Failures in the Technique of Secondary Teaching of Mathematics: Their Causes and Remedies.
William Betz, East High School, Rochester, N. Y., Chairman.
Miss M. E. Shea, High School for Girls, Philadelphia, Pa.
Miss Mary M. Wardwell, Central High School, Buffalo, N. Y.
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Committee No. IV. Mathematics in the Private General Secondary Schools.[1]

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S. A. Courtis, Detroit Home and Day School, Detroit, Mich.
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George P. Tibbetts, Williston Seminary, East Hampton, Mass.
E. J. Owen, Pillsbury Academy, Owatonna, Minn.
Laura A. Whyte, Miss Porter's School, Farmington, Conn.

The following reports, included in the same bulletin as the reports of Committees Nos. III and IV, relate to institutions which, though not secondary schools exclusively, cover more or less of the secondary field in their work.

a. Report on Mathematics in Evening Technical Schools.
By A. D. Dean, State Education Department, Albany, N. Y.

b. Report on Private Correspondence Schools.
By W. F. Rocheleau, Interstate School of Correspondence, Chicago, Ill.

c. Report on Mathematics in Schools and Colleges for Negroes.
By W. T. B. Williams, Hampton Institute, Hampton, Va.

Committee No. V. The Training of Teachers of Elementary and Secondary Mathematics in the United States.[2]

Dr. E. H. Taylor, State Normal School, Charleston, Ill., Chairman.
Professor Clifford B. Upton, Teachers College, Columbia University, New York, N. Y.
H. Clay Harvey, State Normal School, Kirksville, Mo.
Professor M. E. Bogarte, Normal University, Valparaiso, Ind.
J. C. Brown, Horace Mann School, New York, N. Y.

[1] Same bulletin as Committee No. III. [2] Bulletin, 1911, No. 12.

Subcommittee 1. The Training of Teachers of Mathematics in Professional Schools of Collegiate Grade, separate from or connected with Colleges or Universities.

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Professor R. D. Bohannan, Ohio State University, Columbus, Ohio.
Professor A. L. Candy, University of Nebraska, Lincoln, Nebr.
Professor C. D. Rice, University of Texas, Austin, Tex.
Professor L. G. Weld, State University of Iowa, Iowa City, Iowa.

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J. C. Stone, State Normal School, Montclair, N. J.
W. A. Cornish, State Normal School, Cortland, N. Y.
G. C. Shutts, State Normal School, Whitewater, Wis.
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Thomas W. Keenan, Shenandoah, Iowa.
Albert J. Harpman, Austin, Minn.

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J. H. Scarborough, State Normal School, Warrensburg, Mo.
Sands Wright, Teachers College, Cedar Falls, Iowa.

Committee No. VI. Mathematics in the Technical Secondary Schools in the United States.[1]

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J. E. Downey, High School of Commerce, Boston, Mass.
Professor J. E. Ostrander, Massachusetts Agricultural College, Amherst, Mass.

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E. C. Barker, Polytechnic High School, Los Angeles, Cal.
E. R. Smith, Brooklyn Polytechnic Preparatory School, Brooklyn, N. Y.

[The report of this committee is incorporated in the general report.]

Subcommittee 2. Public and Private Commercial Schools.

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Dr. J. T. Rorer, William Penn High School, Philadelphia, Pa.
Raymond G. Laird, Jamaica Plains, Mass.
Miss Clara F. Eaton, New York, N. Y.
Ralph D. Beetle, Dartmouth College, Hanover, N. H.

[1] Bulletin, 1912, No. 4.

Subcommittee 3. Agricultural Schools.

Professor J. E. Ostrander, Massachusetts Agricultural College, Amherst, Mass., Chairman.

Dr. D. L. Stevens, West Raleigh, N. C.

Dr. Tait Butler, Farm Gazette, Starkville, Miss.

Supplementary Report. The Industrial School of Secondary and Intermediate Grade.

Nathan N. Dickler, Manual Training High School, Brooklyn, N. Y.

Committee No. VII. Examinations in Mathematics Other than Those Set by the Teacher for His Own Classes.[1]

Professor T. S. Fiske, Columbia University, New York, N. Y., Chairman.

Principal H. C. Pearson, Horace Mann School, New York, N. Y.

Professor H. D. Thompson, Princeton University, Princeton, N. J.

Professor Virgil Snyder, Cornell University, Ithaca, N. Y.

Dr. C. F. Wheelock, State education department, Albany, N. Y.

Professor N. F. Davis, Brown University, Providence, R. I.

President Robert J. Aley, University of Maine, Orono, Me.

Dr. J. K. Gore, president of the Actuarial Society of America, Newark, N. J.

Subcommittee 1. Nature of Promotion in Elementary Schools and Admission to Secondary Schools.

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H. B. Loomis, Hyde Park High School, Chicago, Ill.

John S. Rackliffe, Huntington School, Brockton, Mass.

R. M. Sherrard, Subdistrict School, Pittsburgh, Pa.

Professor A. Duncan Yocum, University of Pennsylvania, Philadelphia, Pa.

Dr. C. W. Stone, Farmville, Va.

Subcommittee 2. Entrance to College by College Examinations.

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Professor J. L. Coolidge, Harvard University, Cambridge, Mass.

Professor Julius Sachs, Teachers College, Columbia University, N. Y.

Professor L. S. Hulburt, Johns Hopkins University, Baltimore, Md.

Principal J. B. Schobinger, The Harvard School, Chicago.

Subcommittee 3. Entrance to College by College Entrance Board Examinations.

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Miss C. A. Hart, Wadleigh High School, New York, N. Y.

Subcommittee 4. Entrance to College by State Examinations.

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Supt. F. D. Boynton, Ithaca, N. Y.

Subcommittee 5. Entrance to College by Certification.

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Professor George C. Edwards, University of California, Berkeley, Cal.

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Professor William M. Thornton, University of Virginia, Charlottesville, Va.

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[1] Bulletin, 1911, No. 8.

Subcommittee 6. State and Local Examination of Teachers.

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J. C. Byrnes, Board of Education, New York, N. Y.
Harry English, High School, Washington, D. C.

Subcommittee 7. Examination of Actuaries.

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Dr. H. J. Messenger, actuary of the Travelers Insurance Company, Hartford, Conn.
Dr. William M. Strong, assistant actuary of the Mutual Life Insurance Company of New York, New York City.
Mr. Dow, actuary of the New England Mutual Life Insurance Company.

Committee No. VIII. Influences Tending to Improve the Work of the Teacher in Mathematics.[1]

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Professor F. N. Cole, Columbia University, New York, N. Y.
Professor Gustave Legras, College of the City of New York, New York, N. Y.
Dr. A. W. Stamper, State Normal School, Chico, California.
Professor E. B. Skinner, University of Wisconsin, Madison, Wis.
Dr. L. L. Jackson, New York, N. Y.
Professor H. E. Slaught, University of Chicago, Chicago, Ill.

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Subcommittee 6. The Teaching of Mathematics in Summer Sessions of Universities and Normal Schools.

Professor H. E. Slaught, University of Chicago, Chicago, Ill.

[1] Bulletin, 1912, No. 13.

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President C. S. Howe, Case School of Applied Science, Cleveland, Ohio.

Professor S. E. Slocum, University of Cincinnati, Cincinnati, Ohio.

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Committee No. X. Undergraduate Work in Mathematics in Colleges of Liberal Arts and Universities.[2]

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Professor James W. Glover, University of Michigan, Ann Arbor, Mich.

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Professor C. P. Echols, United States Military Academy, West Point, N. Y.

Professor W. J. King, United States Naval Academy, Annapolis, Md.

Wm. E. Breckenridge, Stuyvesant High School, New York, N. Y.

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Professor C. P. Echols, United States Military Academy, West Point, N. Y., Chairman.

Major Tracy C. Dickson, Ordnance Department, Sandy Hook, N. J.

Captain Alston Hamilton, Artillery School, Fort Monroe, Va.

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[1] Bulletin, 1911, No. 9. [2] Bulletin, 1911, No. 7. [3] Bulletin, 1912, No. 2.

Subcommittee 2. The Training of Naval Officers, Including Schools for Graduates of Annapolis.

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Professor E. J. Yowell, Annapolis, Md.
Professor C. H. Sisam, Urbana, Ill.

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Professor D. R. Curtiss, Northwestern University, Evanston, Ill.
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Professor H. E. Hawkes, Columbia University, New York, N. Y.
Professor G. D. Olds, Amherst College, Amherst, Mass.

[1] Bulletin, 1911, No. 6.

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NOTE.—The documents indexed herein are as follows:

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Committees Nos. III and IV.—Mathematics in the Public and Private Secondary Schools of the United States. (Bulletin, 1911, No. 16.)

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Committee No. XII.—Graduate Work in Mathematics in Universities and in Other Institutions of Like Grade in the United States. (Bulletin, 1911, No. 6.)

Abbreviations used: Am. Com. = Report of the American Commissioners. *13-1911*, I and II = Bulletin No. 13, 1911, Committees I and II, etc.

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